
MATHCOUNTS

1989-90
■ State Competition ■
Sprint Round

Name _____

School _____

Chapter _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This booklet contains 30 questions. You will have 40 minutes to complete all the questions. Calculators, slide rules, books, or any other aids are not permitted to be used during the contest. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blank in the right column of the contest booklet. If you complete the round before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the Cray Research Foundation, the General Motors Foundation, the National Council of Teachers of Mathematics, the National Aeronautics and Space Administration, and the United States Department of Education.

1. Suzie's dog eats $2\frac{1}{2}$ cups of dog food per day. If the dog food weighs 4 ounces per cup, how many days can Suzie feed her dog with a 40 pound bag? 1. _____

2. How many thirty-sixths are in $83\frac{1}{3}\%$? 2. _____

3. Find the result, in decimal form, when 1.25% of 1000 is added to 12.5% of 100 and that sum is subtracted from 125% of 10. 3. _____

4. Evaluate this expression if $x = -3$, $y = 5$ and $z = -2$. 4. _____

$$z - 5(z(x - y) - 12x)$$

5. If the greatest common factor (GCF) of a and b is 1, what is the least common multiple (LCM) of $12a^2b$ and $30ab^2$? 5. _____

6. Two cards are drawn one at a time from a standard deck of 52 cards. What is the probability, expressed as a fraction, that both are kings? 6. _____

7. A recent survey of voters regarding an environmental issue on the ballot is summarized in the following table: 7. _____

	<u>In Favor</u>	<u>Opposed</u>
Number of Females	348	469
Number of Males	482	326

What percent (to the nearest whole number percent) of the voters surveyed were female or in favor of the issue?

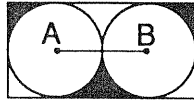
8. Solve for z:

$$\frac{z + 3}{z + 5} = \frac{z - 2}{z - 5}$$

8. _____

9. Find the area of the shaded portion of the rectangle, where $AB = 9$. Express the answer in terms of π .

9. _____



10. If b and 7 are solutions to the equation: $x^2 - 4x = a$, find the value of $a + b$.

10. _____

11. If the radius of a circle is decreased by 20%, by what percent is the area decreased?

11. _____

12. A triangle has vertices with coordinates $A(0,15)$, $B(0,0)$, and $C(10,0)$. Find the coordinates of point D on AC so that the area of triangle ABD is equal to the area of triangle DBC.

12. _____

13. If an iron ball 8 inches in diameter weighs 80 pounds, how many pounds does an iron ball 12 inches in diameter weigh?

13. _____

14. Find the simplest common fraction equivalent to .037.

14. _____

15. If $y = \frac{3z - z^3}{2}$ and $z = x^2 + 2$, what is y to the nearest integer when x is 0.5? 15. _____

16. Kathi earns \$3.55 per hour and receives a \$.15 per hour pay raise every 6 months. Jack earns \$6.05 per hour and has a pay decrease of \$.10 per hour every 6 months. How many years will it be until they earn the same hourly wage? 16. _____

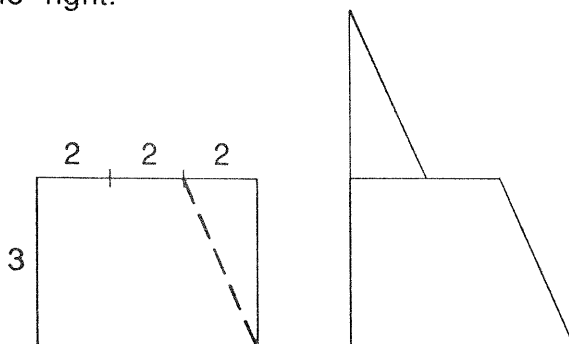
17. Find all integer values of x for which the equation $(x + 3)(x - 2) < 0$ is true. 17. _____

18. A formula is programmed into a computer. It results in the following: 18. _____

Input:	4	5	6	10
Output:	23	32	43	107

If the same formula is used, what would be the output value if 14 is entered?

19. The rectangular region shown on the left is cut on the dotted line and reassembled to form the figure on the right. What is the area of the figure shown on the right? 19. _____



20. Express the following as a simple, reduced, rational number.

20. _____

$$2 - \frac{1}{1 + \frac{2}{1 + \frac{1}{2}}}$$

21. Two typists working together can type a paper in 5 hours. The faster typist can complete the paper in 9 hours. How many hours will it take the slower typist to complete the paper alone? (Express your answer as a mixed number.)

21. _____

22. Express the sum as a base ten numeral:

22. _____

$$11_{\text{two}} + 11_{\text{three}} + 11_{\text{four}} + \dots + 11_{\text{twelve}}$$

23. Find the length of the line segment with endpoints (a,a) and (9a, 7a) and $a > 0$.

23. _____

24. A manufacturing company identifies its products by codes consisting of a letter (A-Z) followed by 3 digits (0-9) followed by another letter. How many different products can the company manufacture before it uses up all possible codes?

24. _____

25. Find the sum of the first 14 terms of the arithmetic series: $2 + 5 + 8 + 11 + \dots$

25. _____

26. The first four gymnastics judges gave Mary scores of 9.25, 9.4, 9.3, and 9.45. What score, expressed as a decimal number, did she need from the fifth judge to get an average score of 9.45? 26. _____

27. It took 24 workers working 10 hours per day for 24 days to complete a task. At this rate, how many days will it take 60 workers to complete the same task working 8 hours per day? 27. _____

28. The eighth grade basketball team averaged P points per game in the X games it won this season. The team scored a total of T points during the season. Express the total number of points scored in the games it lost in terms of P , X , and T . 28. _____

29. K is the smallest positive integer satisfying these properties. 29. _____

When K is divided by 5 the remainder is 4.
When K is divided by 8 the remainder is 2.
When K is divided by 11, the remainder is 1.
 K is _____

30. The area of a semicircular region is $\frac{49}{8}\pi$ square inches. Express the number of inches in the perimeter of the semicircular region to the nearest whole number. 9.25, 9.4, 9.3, and 9.45. What score, expressed as a decimal number, did she need from the fifth judge to get an average score of 9.45? 30. _____