MATHCOUNTS

1991-92

■ State Competition ■

Sprint Round

Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

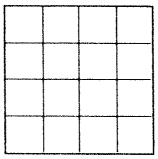
This booklet contains 30 questions. You will have 40 minutes to complete all the questions. You are not allowed to use calculators, slide rules, books, or any other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blank in the right column of the contest booklet. If you complete the round before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the Cray Research Foundation, the General Motors Foundation, the National Council of Teachers of Mathematics, and the National Aeronautics and Space Administration.

1. A sequence of numbers is formed using the rule $\frac{n^2 + n}{2}$ for $n = 1, 2, 3, \ldots, 10$. What percent of the terms of the sequence are perfect squares? 1. _____

2. If only one X can be placed in each square, what is the maximum number of X's that can be placed in the 4-by-4 grid of squares so that no row, column, or diagonal contains 4 X's? 2. _____



3. If n is any positive integer, what is the smallest possible value of $n + \frac{16}{n}$? 3. _____

4. A rectangle 10 cm long and 8 cm wide is to be enlarged proportionately so that its area is increased by 44%. Find the length, in centimeters, of the new rectangle. 4. _____

5. In what digit does 3^{571} end? 5. _____

6. A, B, C, D, and E are consecutive points on a line. If $\frac{AB}{BC} = \frac{1}{3}$, $\frac{BC}{CD} = \frac{1}{4}$, and $\frac{CD}{DE} = \frac{1}{2}$, what is $\frac{AC}{BE}$? Express your answer as a common fraction. 6. _____

7. A fungus culture growing under ideal conditions doubles in size each day. How many units will the culture contain after four days if it originally contained 10 units? 7. _____

8. Solve for x: $9^2 = 27^{2x \cdot 1}$. Express your answer as a common fraction. 8. _____

9. The mean of 21 test scores is 78. What mean score must three additional students achieve so that the mean for all 24 students is 80? 9. _____

10. Julie begins counting backward from 1000 by 2's, and, at the same time, Tony begins counting forward from 100 by 3's. If they count at the same rate, what number will they say at the same time? 10. _____

11. When 24 chocolates are put in a box that weighs 3 ounces empty, the total weight is $1\frac{1}{2}$ pounds. What will be the total weight if four chocolates are removed? Express your answer as a mixed number of pounds. 11. _____

12. In a certain country, 20% of the people possess 80% of the wealth. How many times wealthier is the average person in the smaller group than the average person in the remaining 80% of the population? 12. _____

13. A circle is divided into twenty equal sectors. How many adjacent sectors combined would form an angle of 72 degrees? 13. _____

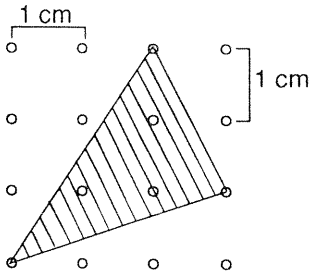
14. Find the sum of all integers that satisfy these conditions: 14. _____

$|x| + 1 > 7$ and $|x + 1| \leq 7$.

15. What is the average rate, in miles per hour, of a car that travels 20 miles at 40 miles per hour and then 10 miles at 60 miles per hour? 15. _____

16. Triangle ABC is an equilateral triangle and O is the center of its inscribed circle. If the area of the circle is 4π cm^2, what is the area, in square centimeters, of triangle ABC? Express your answer in simplest radical form. 16. _____

17. In the square grid below, what is the area, in square centimeters, of the shaded region? Express your answer as a mixed number. 17. _____



18. The sum of 101 consecutive integers is 101. What is the largest integer in the sequence? 18. _____

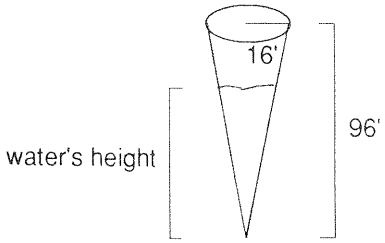
19. If a prism has 12 vertices, how many edges does it have? 19. _____

20. Express $0.\overline{1} + 0.\overline{01} + 0.\overline{0001}$ as a common fraction. 20. _____

21. In how many ways can 9 be written as the sum of three (not necessarily distinct) positive integers if order is not important? 21. _____

22. Robert has twice as many dimes as quarters. If the number of dimes and quarters is reversed, by what percent does the value of his money increase? Express your answer as a mixed number. 22. _____

23. The water tank in the diagram below is in the shape of an inverted right circular cone. The radius of its base is 16 feet, and its height is 96 feet. What is the height, in feet, of the water in the tank if the amount of water is 25% of the tank's capacity? Express your answer in simplest radical form. 23. _____



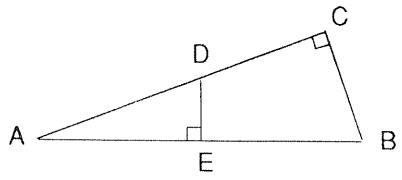
24. Two numbers x and y have a geometric mean of 12 and an arithmetic mean of 12.5. Find $x^2 + y^2$. 24. _____

25. Right triangle ABC with right angle at vertex C has median $\overline{AD}$ of length 5 and median $\overline{BE}$ of length $2\sqrt{10}$. Find the length of the hypotenuse of triangle ABC. Express your answer in simplest radical form. 25. _____

26. Find the remainder when 2^{100} is divided by 5. 26. _____

27. What is the degree measure of the acute angle formed by the hands of a clock at 3:20? 27. _____

28. In the figure, AE = 6, EB = 7, and BC = 5. What is the area of quadrilateral EBCD ? Express your answer as a common fraction. 28. _____



29. A topless cardboard box, 5 cm by 5 cm by 5 cm, is filled with 1-cm-by-1-cm-by-1-cm cubes. How many of the cubes touch cardboard? 29. _____

30. Digit **d** is randomly selected from the set {4,5,6,7}. Without replacement of **d**, another digit **e** is selected. What is the probability that the two-digit number **de** is a multiple of 3? Express your answer as a common fraction. 30. _____