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# MATHCOUNTS

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1992-93

■ Chapter Competition ■  
Sprint Round

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Name \_\_\_\_\_

School \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This booklet contains 30 questions. You will have 40 minutes to complete all the questions. You are not allowed to use calculators, slide rules, books, or any other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blank in the right column of the competition booklet. If you complete the round before time is called, use the remaining time to check your answers.

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| Total Correct | Scorer's Initials |
|---------------|-------------------|
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MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the Cray Research Foundation, the General Motors Foundation, Texas Instruments Incorporated, the National Council of Teachers of Mathematics, and the National Aeronautics and Space Administration.

1. What part of  $\frac{1}{2}$  of a square meter is  $(\frac{1}{2} \text{ meter})$  squared? Express your answer as a common fraction. 1. \_\_\_\_\_
  
2. If  $a * b = \frac{1}{a} + \frac{1}{b}$ , for what decimal value of  $a$  is  $a * 0.2 = 10$ ? 2. \_\_\_\_\_
  
3.  $\sqrt{50} + \sqrt{75}$  lies between two consecutive integers,  $a$  and  $b$ , where  $a < b$ . What is the value of  $b$ ? 3. \_\_\_\_\_
  
4. A number is chosen at random from the following set:  $\{ 0.25, 0.5, 0.75, 0.8, 1, 2, 2.2, 3, 4, 9.7 \}$ . What is the probability that its reciprocal is greater than 1? Express your answer as a common fraction. 4. \_\_\_\_\_
  
5. How many integers belong to the arithmetic sequence 2, 5, 8, 11, ..., 302? 5. \_\_\_\_\_
  
6. Two boxes contain a total of 150 coins. 17 coins are moved from the first box to the second. The first box now contains half as many coins as the second. How many coins were originally in the first box? 6. \_\_\_\_\_
  
7. Two trains are headed directly towards each other at rates of 80 mph and 120 mph. How many miles apart are they 1 minute before impact? Express your answer as a mixed number. 7. \_\_\_\_\_

8. A school band found they could arrange themselves in rows of 6, 7, or 8 with no one left over. What is the minimum number of students in the band?

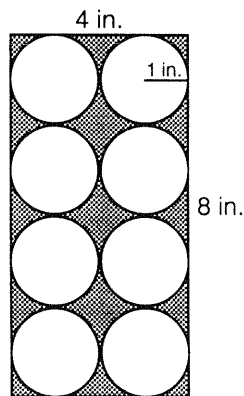
8. \_\_\_\_\_

9. The number 13 is prime. If you reverse the digits you also obtain a prime number, 31. What is the larger of the pair of primes that satisfies this condition and has a sum of 110?

9. \_\_\_\_\_

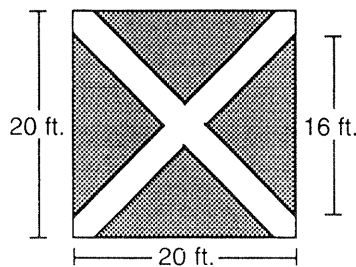
10. Eight circular cookies are cut from a rectangle of dough measuring 4 inches by 8 inches. How many square inches of dough are left over? Express your answer in terms of  $\pi$ .

10. \_\_\_\_\_



11. A garden is laid out in the fashion shown in the diagram. If only the shaded isosceles triangles are used for planting, what is the total area, in square feet, that is to be used for planting?

11. \_\_\_\_\_



12. Find the remainder when  $1992^2$  is divided by 9.

12. \_\_\_\_\_

13. Cards are randomly drawn one at a time without replacement from a standard deck of playing cards. What is the probability, expressed as a common fraction, that the first three cards chosen are clubs? 13. \_\_\_\_\_

14. What is the 17th digit after the decimal point in the decimal expansion of  $\frac{11}{7}$ ? 14. \_\_\_\_\_

15. If  $a$  and  $b$  are numbers such that  $a + b = 18$ , what is the smallest value of  $a^2 + b^2$ . 15. \_\_\_\_\_

16. Compute the quotient  $\frac{(17^2 - 10^2)}{(17 - 10)}$ . 16. \_\_\_\_\_

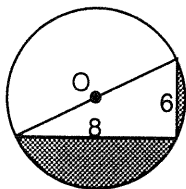
17. Find the sum of the reciprocals of all the factors of 24. Express your answer as a decimal. 17. \_\_\_\_\_

18. If light travels at a speed of  $3.0 \times 10^5$  km/sec and the earth is approximately 150,000,000 kilometers from the sun, to the nearest second, how many seconds does it take light from the sun to reach the earth? Express your answer in scientific notation. 18. \_\_\_\_\_

19. The length of a side of a cube is 8 units. Express, as a common fraction, the ratio of the number of square units in the surface area of the cube to the number of cubic units in its volume. 19. \_\_\_\_\_

20. The value of an automobile that originally sells for \$16,000 loses 10% of its value the first year and 5% of the original value each year thereafter. In which year will the value of the car fall below \$6,000? 20. \_\_\_\_\_
21. The diagonal of a cube is 5 inches. How many square inches are in the total surface area of the cube? 21. \_\_\_\_\_
22. Let  $\frac{a \backslash b / c}{f \backslash e / d}$  be defined as  $ad + be + cf$ . If  $\frac{2 \backslash 3 / 4}{6 \backslash e / 5}$  equals 67 then what must e equal? 22. \_\_\_\_\_
23. If  $(x + 2)(3x^2 - x + 5) = Ax^3 + Bx^2 + Cx + D$ , what is the value of  $A + B + C + D$ ? 23. \_\_\_\_\_
24. Jane's mother asked her to drain and refill the fish pond in the backyard. It takes five hours for the pond to empty. The hose will refill it in three hours. Unfortunately, Jane did not close the drain when she turned on the hose. Assuming the pond fills and drains at a constant rate, how many hours did it take to refill the pond? Express your answer as a decimal. 24. \_\_\_\_\_
25. How many distinct arrangements of the letters in the word CHAIR are possible? 25. \_\_\_\_\_
26. The arithmetic mean of a set of ninety numbers is 47. One-third of the numbers are increased by six. What is the mean of the new set? 26. \_\_\_\_\_

27. Express the area of the shaded region in circle O in terms of  $\pi$ . 27. \_\_\_\_\_



28. Find the coordinate of the point halfway between  $-1\frac{5}{6}$  and  $\frac{3}{5}$  on the number line. Express your answer as a mixed number. 28. \_\_\_\_\_

29. A dime, 2 nickels, and 3 pennies are in a container. Assume that it is equally likely to shake out any one coin. What is the probability of shaking out a penny each of 4 times if the coin is returned after each shake? Express your answer as a common fraction. 29. \_\_\_\_\_

30. Using 2.24 as an approximate value of  $\sqrt{5}$ , find the approximate value of  $\frac{1}{\sqrt{5}}$  to the nearest hundredth. 30. \_\_\_\_\_