
MATHCOUNTS

1992-93

■ State Competition ■
Sprint Round

Name _____

School _____

Chapter _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This booklet contains 30 questions. You will have 40 minutes to complete all the questions. You are not allowed to use calculators, slide rules, books, or any other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blank in the right column of the competition booklet. If you complete the round before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

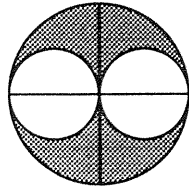
MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the Cray Research Foundation, the General Motors Foundation, Texas Instruments Incorporated, the National Council of Teachers of Mathematics, and the National Aeronautics and Space Administration.

1. Jill climbed a hill at a uniform rate. At 2 PM she was $\frac{1}{4}$ of the way up and at 3 PM she was $\frac{2}{3}$ of the way up. How many minutes before 2 o'clock did Jill begin her climb?

1. _____

2. Two congruent circles are inscribed in the large circle. The radius of the large circle is the same as the diameter of either small circle. What is the ratio of the area of the shaded region to the area of the large circle? Express your answer as a common fraction.

2. _____



3. Find the sum of the values of x which satisfy $x^2 + 1992x = 1993$.

3. _____

4. Given $(\frac{x}{3} - \frac{3}{x})^2 = 0$, find the value of x^4 .

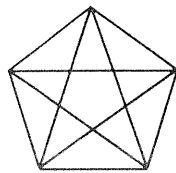
4. _____

5. Of 10 boxes, 5 contain pencils, 4 contain pens, and 2 contain both pens and pencils. How many boxes contain neither pens nor pencils?

5. _____

6. How many triangles are in this figure?

6. _____



7. How many ordered integral triples (a, b, c) have the property that each number is the product of the other two?

7. _____

8. Randomly choosing two numbers from the set $\{1, 3, 5, 7, 9\}$ with replacement, what is the probability that the product is greater than 40? Express your answer as a percent.

8. _____

9. A book contains 250 pages. How many times is the digit 2 used in numbering the pages? 9. _____

10. A book is opened and the product of the two page numbers that appear is 3906. What is the larger of the two page numbers? 10. _____

11. Twenty students hold a chess tournament in which each of the students will play every other student in the group once. How many individual contests will be held? 11. _____

12. The product of two numbers has been accidentally erased by a student. All that remains is shown here. The student only remembers that the product was larger than 4000. Name the larger factor. 12. _____

$$\begin{array}{r} _ \ 8 \\ \times _ _ \\ \hline 3 _ _ \\ _ \ 1 \ 2 \\ \hline _ _ _ _ \end{array}$$

13. Of the 30 students in Ms. Smith's class, 14 are boys. There are 13 students who play a musical instrument and six of these are boys. How many girls in the class do not play a musical instrument? 13. _____

14. Determine the units digit of $17^{13} - 17$. 14. _____

15. Pat has \$1.13 in pennies, nickels, and quarters. If she has no more than 4 coins of each type, how many coins does she have altogether? 15. _____

16. Rick's Yogurt Shop has scoops of three sizes: 2 inch, $2\frac{1}{2}$ inch and 3 inch diameter which sell for \$1, \$2, and \$3 respectively. Which size scoop is the worst buy? 16. _____

17. The line $y = ax + b$ is perpendicular to the line $y = 2x - 3$ and passes through the point (0,4). What is $a + b$? Express your answer as a mixed number. 17. _____

18. What is the maximum number of points of intersection when two circles and three straight lines intersect each other? Assume no figure coincides with another. 18. _____

19. The ratio of two integers is $\frac{5}{3}$. If 10 is subtracted from each integer, the ratio is $\frac{5}{2}$. What is the sum of the two integers in the original ratio? 19. _____

20. If x , y , and z are positive integers such that $x^2 + y^2 + z^2 = 174$, what is the greatest possible value of $x + y + z$? 20. _____

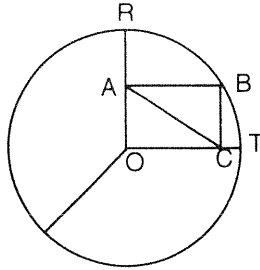
21. What is the volume in cubic inches of a rectangular box that has sides of areas 48, 66, and 88 square inches? 21. _____

22. Suppose a , b , and c are 1, 2, and 3 in some order. What is the largest possible sum that could result from the problem below? 22. _____

$$\begin{array}{r} a c b 1 \\ 2 b a c \\ + c b 3 a \\ \hline \end{array}$$

23. If x and y are positive numbers less than 20 for which $x + y + xy = 76$, what is $x + y$? 23. _____

24. In the figure below, O is the center of the circle and point B is on the circle. given $OR = 8$ and $CT = 2$, find the length of diagonal \overline{AC} in rectangle ABCO.



24. _____

25. The sum of the number of cubic units in the volume plus the number of square units in the surface area of a cube equals six times the number of units in the sum of the lengths of its edges. What is the length of each edge of the cube?

25. _____

26. How many distinct isosceles triangles having sides of integral length and perimeter 113 are possible?

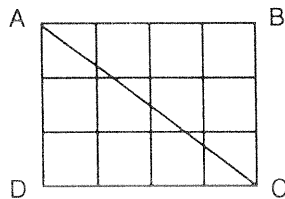
26. _____

27. How many square centimeters are in the surface area of a regular octahedron which has edges of 4 cm?

27. _____

28. Rectangle ABCD has been divided into 12-unit squares as shown. Through how many squares (of any size) does segment AC pass?

28. _____



29. Together Pam, Dan, and Sam have \$31.00. How many dollars does Sam have if Dan has \$3.00 less than Pam, and Sam has \$1.00 more than Pam and Dan combined?

29. _____

30. Evaluate: $(\sqrt[3]{0.000027} \cdot \sqrt[3]{0.064})^2$

30. _____