

---

---

# MATHCOUNTS

---

1993-94

■ Chapter Competition ■  
Sprint Round

---

Name \_\_\_\_\_

School \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO**

This booklet contains 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, slide rules, books, or any other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blank in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

---

Total Correct	Scorer's Initials

MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the Cray Research Foundation, the General Motors Foundation, the Intel Foundation, Texas Instruments Incorporated, the National Council of Teachers of Mathematics, and the National Aeronautics and Space Administration.

1. The spindle speed of a CD player is 200 revolutions per minute. How many revolutions does the spindle make while playing a song that lasts exactly 3 minutes?

1. \_\_\_\_\_

2. A tanker carries 10,000 gallons of gasoline. Gasoline weighs 5.99 pounds per gallon at 60 degrees Fahrenheit and 6.25 pounds per gallon at zero degrees Fahrenheit. How many pounds heavier is the tanker when it carries a full load at the colder temperature than at the warmer temperature?

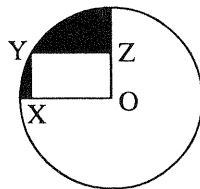
2. \_\_\_\_\_

3. Simplify:  $3(\frac{1}{6} + \frac{1}{2}) + 2$ .

3. \_\_\_\_\_

4. In the figure,  $XO = 8$ ,  $ZO = 6$ , and the vertex Y of rectangle XYZO lies on circle O. Find the area of the shaded region. Express your answer in terms of  $\pi$ .

4. \_\_\_\_\_



5. Express the number of square units in the total surface area of a cube in terms of its interior diagonal,  $d$ .

5. \_\_\_\_\_

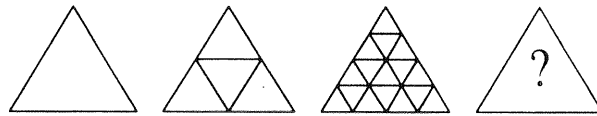
6. For what value of  $n$  will  $(7!)(5!)(3!)(1!) = n!$ ?

6. \_\_\_\_\_

7. In a seven-story building on which floor would you be if the walk to the seventh floor would be twice as far as the walk to the first floor? 7. \_\_\_\_\_

8. Twenty-six people, numbered consecutively 1 through 26, are seated equally spaced around a circular table. Which numbered person is seated directly across from person number 9? 8. \_\_\_\_\_

9. By continuing the pattern below, how many non-overlapping triangles would appear in the last figure? 9. \_\_\_\_\_



10. What is the least common multiple of 45, 60, and 75? 10. \_\_\_\_\_

11. When the repeating decimal  $0.\overline{12}$  is expressed as a common fraction in lowest terms, what is the sum of its numerator and denominator? 11. \_\_\_\_\_

12. How many cubic centimeters are in the volume of a cone having a diameter of 10 cm and a height of 6 cm? Express your answer in terms of  $\pi$ . 12. \_\_\_\_\_

13. Of the following, which is the best buy, A or B?

A) 10 lbs of apples for \$3.95

B) 5Kg of apples for \$3.85

13. \_\_\_\_\_

14. How many integers between 100 and 500 do not contain an even digit?

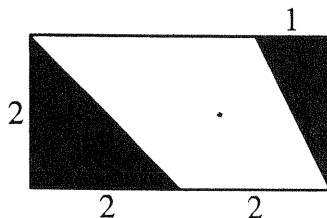
14. \_\_\_\_\_

15. Out of their first 40 games the Williamsburg Wildwings had a record of 18 wins and 22 losses. Assuming no ties, how many of their next 50 games must they win if their record is to show twice as many wins as losses?

15. \_\_\_\_\_

16. What fraction of the rectangle is shaded? Express your answer as a common fraction.

16. \_\_\_\_\_



17. A tennis racquet originally sold for \$120.00. For a special sale the price was reduced by 10%. After the sale it was then marked up 10%. What was the price of the tennis racquet after the sale?

17. \_\_\_\_\_

18. What reduced common fraction is equivalent to  $20\frac{5}{6}\%$  ?

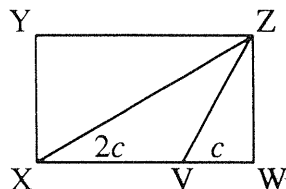
18. \_\_\_\_\_

19. If  $x \clubsuit y = (x^y)^x$ , what is the units digit of  $4 \clubsuit 3$ ?

19. \_\_\_\_\_

20. Find the ratio of the area of triangle XZV to the area of the rectangle XYZW.

20. \_\_\_\_\_



21. Express as a common fraction the product of:

21. \_\_\_\_\_

$$\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \cdots \left(1 - \frac{1}{100}\right).$$

22. The harmonic mean,  $m$ , of  $a$  and  $b$  is given by  $\frac{2}{\frac{1}{a} + \frac{1}{b}}$ .  
What is the harmonic mean of 3 and 6?

22. \_\_\_\_\_

23. Susan's calculator has a key which replaces the number displayed by its cube. If a 2 is displayed, how many times must Susan press the "cubing key" to display a number which is greater than a billion?

23. \_\_\_\_\_

24. The Golden Mean,  $g$ , is a positive number that is one more than its reciprocal. Find  $g$  to the nearest hundredth.

24. \_\_\_\_\_

25. Given  $\boxed{a \ b} = ab - a - b$ , find the value of  $b$  in the equation  $\boxed{3 \ b} = 5$ . 25. \_\_\_\_\_

26. Simplify  $(22 + 2.22) \div 0.2$ . Express your answer as a decimal. 26. \_\_\_\_\_

27. An ordered pair  $(a, b)$  is determined by choosing a number  $a$  and then a number  $b$  at random without replacement from the set  $\{1, 2, 3, 4, 5\}$ . What is the probability that  $\frac{a}{b}$  is an integer? Express your answer as a common fraction. 27. \_\_\_\_\_

28. Solve for  $x$ :  $(2^5 - 2^3)(2^4 - 2^2) \div 9 = 2^{x+1}$ . 28. \_\_\_\_\_

29. Seventy-five tickets to a student performance were sold. Adult tickets sold for \$12.50 and student tickets sold for \$8.50. If the income from the ticket sales was \$865.50, how many adult tickets were sold? 29. \_\_\_\_\_

30. How many integers,  $x$ , satisfy  $|5x - 3| \leq 7$ ? 30. \_\_\_\_\_