
MATHCOUNTS

1993-94

■ State Competition ■
Sprint Round

Name _____

School _____

Chapter _____

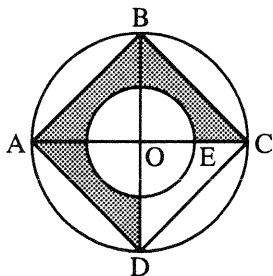
**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO**

This booklet contains 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, slide rules, books, or any other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blank in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the Cray Research Foundation, the General Motors Foundation, the Intel Foundation, Texas Instruments Incorporated, the National Council of Teachers of Mathematics, and the National Aeronautics and Space Administration.

1. In the diagram both circles have center O, \overline{AC} and \overline{BD} are diameters of the larger circle, $\overline{AC} \perp \overline{BD}$, $AO = 2$, and $OE = 1$. Find the area of the shaded region. Express your answer as a common fraction in terms of π .



1. _____

2. A fair, 8-sided die has faces numbered 1, 2, 3, 4, 5, 6, 7, and 8. If the die is rolled twice, what is the probability that the sum of the two numbers will be 9? Express your answer as a common fraction.

2. _____

3. A single large banquet table is created by joining the sides of several small square tables, each of which seats one person for each open side. What is the maximum number of people that could be seated at a table which uses 10 small tables?

3. _____

4. An 8 inch by 10 inch rectangular picture is surrounded by a border that is 2 inches wide on all sides. Find the ratio of the area of the border to the area of the picture. Express your answer as a common fraction.

4. _____

5. If $\frac{1}{x} + \frac{1}{y} = 3$ and $\frac{1}{x} - \frac{1}{y} = -7$ what is the value of $x + y$? Express your answer as a common fraction.

5. _____

6. Every member of a math club is taking algebra or geometry and 8 are taking both. If there are 17 taking algebra and 13 taking geometry, how many members are in the club? 6. _____
7. Two factors have a product of $3 \cdot 9^{2x} - 6 \cdot 27^x$. One of the factors is $3^x - 2$. What is the other factor? 7. _____
8. A regular hexagon of side 7 is surrounded on each side by regular hexagons of the same size adjacent to its sides. What is the perimeter of the resulting figure? 8. _____
9. If 2^n is a factor of $20!$, what is the largest possible value of n ? 9. _____
10. A wrecker's iron ball eight inches in diameter weighs 120 pounds. How many pounds would a similar iron wrecking ball twelve inches in diameter weigh? 10. _____
11. A rectangle $XYZW$ has $XY = 6$, and $XW = 8$. If a point M is selected randomly in the interior of this rectangle, what is the probability that triangle XMW has an area equal to or greater than 16 square units? Express your answer as a common fraction. 11. _____

12. Fruit salad can be made with any 3 of these 5 fruits: apples, bananas, grapes, strawberries, and pineapples. If strawberries and pineapples do not taste good together and grapes and bananas do not look appetizing together how many possible good tasting and appetizing salads are there? 12. _____

13. A group of boys and girls reserved a single row of 120 seats at a theater. The girls were assigned seat numbers in such a way that every boy had to sit next to at least one girl. What is the fewest number of girls necessary to make this happen? 13. _____

14. A and B are non-zero digits for which $A468B05$ is divisible by 11. What is $A + B$? 14. _____

15. Point Z is the midpoint of \overline{MN} . Point P is the midpoint of \overline{ZN} . Point Q is the midpoint of \overline{PN} and Point R is the midpoint of \overline{QN} . If \overline{PR} is 24 inches long, how many feet long is \overline{MN} ? Express your answer as a mixed number. 15. _____

16. To the nearest hundredth of a gallon, how many gallons of gasoline can be saved each year by driving a car which gets 32 miles per gallon as opposed to a car which gets 18 miles per gallon? Assume the number of miles traveled each year is 9,000. 16. _____

17. Find the value of K for which these two equations will not have a common solution: 17. _____

$$6x + 4y = 7$$

$$Kx + 8y = 7$$

18. Which of the fractions $\frac{13}{40}$, $\frac{17}{52}$, $\frac{21}{64}$, and $\frac{25}{76}$ is the smallest?

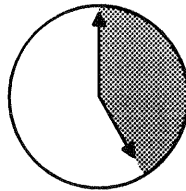
18. _____

19. Massaru has 3 pencils in his bookbag. Two pencils are blue and one is red. If he randomly selects two pencils, what is the probability that they are the same color? Express your answer as a common fraction.

19. _____

20. The hour hand of a broken clock points to 5 and the minute hand to 12. To the nearest percent what part of the clock face is shaded?

20. _____



21. Suppose A and B are non-zero digits and $\overline{A5} \cdot \overline{B1} = 1995$. Find $A + B$.

21. _____

22. A sequence of integers is obtained by starting with three digits d_1 , d_2 , and d_3 . The fourth term of the sequence is the units digit of $d_1 + d_2 + d_3$. Each succeeding term is the units digit of the sum of the three previous terms. What numbers belong in the first, second, and third positions respectively to complete the sequence?

22. _____

____, ____, ____, 1, 1, 1

23. If $A \diamond B = \frac{2A-B}{2}$, what is the value of $(3 \diamond 4) \diamond 5$? Express your answer as a common fraction. 23. _____
24. What integer is closest to $\frac{12.55 \cdot 8 \cdot 61}{24.88 \cdot 82}$? 24. _____
25. How many positive integers have cube roots less than $1 + \sqrt{2}$? 25. _____
26. Find the sum of $\frac{4!}{6!3!} + \frac{3!}{5!}$. Express your answer as a common fraction. 26. _____
27. The sum of the digits of a two-digit number is 12. When the digits are reversed, the original number is increased by 54. What is the product of the digits? 27. _____
28. Out of 22 students surveyed on ice cream flavors, 12 liked chocolate, 5 liked only strawberry, and 6 liked vanilla. If 3 liked chocolate and vanilla, how many students did not like any of these flavors? 28. _____
29. What is the value of x in $222^x - 111^x = 111^x \cdot 7$? 29. _____
30. There exist non-negative integers a and b such that for any x satisfying $a < x < b$, it is true that $\sqrt{x} > x$. Find the ordered pair (a, b) . 30. _____