MATHCOUNTS

1994–95
■ State Competition ■
Sprint Round

Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, slide rules, books, or any other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible, and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

MATHCOUNTS is a cooperative project of the National Society of Professional Engineers, the CNA Insurance Companies, the General Motors Foundation, the Intel Foundation, Texas Instruments Incorporated, the National Council of Teachers of Mathematics, and the National Aeronautics and Space Administration.

1. Two distinct numbers are chosen at random from $\{1, 2, 3, 4, 5, 6\}$. What is the probability that the quotient of the smaller number divided by the larger number is a terminating decimal? Express your answer as a common fraction. 1. _____

2. The notation $a \equiv b \pmod{n}$, where n is a positive number, means $(a - b)$ is a multiple of n. What is the smallest positive integer x such that $3x \equiv 4 \pmod{5}$? 2. _____

3. The radius of a right circular cylinder is decreased by 20% and its height is increased by 25%. What is the absolute value of the percent change in the volume of the cylinder? 3. _____

4. What is the maximum number of points of intersection of two congruent squares that do not share a common line segment? 4. _____

5. An isosceles triangle is inscribed in a circle so that one of its sides is a diameter. The ratio of the area of the triangle to the area of the circle is $1 : a$. Express a in terms of π. 5. _____

6. How many $\frac{1}{2}$-inch cubes are needed to make 1 cubic foot? 6. _____

7. A three-digit number is divided by a two-digit number, yielding an integer quotient and zero remainder. What is the smallest possible integer quotient? 7. _____

8. Find the number of square meters in the area of a regular hexagon inscribed in a circle of diameter 12 meters. 8. _____

9. A 1,000-foot long retaining wall is to be built, partly of wood and the rest of stone. The wood costs \$8 per foot and the stone costs \$10 per foot. The total cost can be at most \$9,200. What is the greatest number of feet of stone that can be used? 9. _____

10. Given that it takes four miles of fence to enclose a square 640-acre field, how many acres are in a square field enclosed by two miles of fence? 10. _____

11. Given an isosceles trapezoid with bases of 8 and 18 and an area of 156 square units, what is the number of units in the length of one of the non-parallel sides? 11. _____

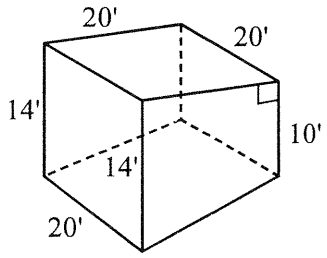
12. Solve for a: $\dfrac{8^{-1}}{4^{-1}} - a^{-1} = 1$ 12. _____

13. Angela, Bob, and Charlie have a total of \$35 in dollar bills. Bob has $\frac{3}{4}$ as much as Angela and Charlie together. How many dollar bills does Bob have? 13. _____

14. Find the sum of the x-coordinates of all possible positive integral solutions to $\frac{1}{x} + \frac{1}{y} = \frac{1}{7}$. 14. _____

15. Given x and y are integers such that $-20 \leq x \leq 10$, $-10 \leq y \leq 20$, $n = \frac{x}{y}$, and $y \neq 0$, find the sum of the largest and smallest possible values of n. 15. _____

16. The diving pool shown is in the shape of a trapezoidal right prism. How many cubic feet are in its volume? 16. _____



17. Solve for x in terms of k: $2^x + 2^x + 2^x + 2^x + 2^x + 2^x = 3 \cdot 2^k$ 17. _____

18. Twenty-seven solid gold spherical beads each of radius 3 are melted down and recast into a larger, solid gold sphere. How many units are in the radius of this larger gold sphere? 18. _____

19. A four-digit number is chosen at random from all four-digit numbers. Express as a common fraction the probability that the number is divisible by 2, 3, 4, and 5. 19. _____

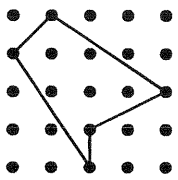
20. A geometric solid is to be built by joining the faces of 26 cubes, each having volume of one cubic inch. What is the positive difference between the number of square inches in the largest and smallest possible surface areas of such a solid? 20. _____

21. Solve for x and express as a common fraction: $\frac{1}{2}(4^{3x}) = 128$ 21. _____

22. A teacher asks for a group of volunteers from a class of 6 students to participate in a class project. Assuming that at least one student volunteers, how many combinations of volunteers are possible? 22. _____

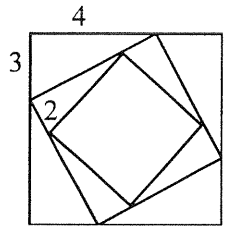
23. Find the value of $\frac{1}{100} + \frac{2}{100} + \frac{3}{100} + \cdots + \frac{100}{100}$. Express your answer as a decimal. 23. _____

24. In this rectangular array, the dots are one inch apart horizontally and vertically. What is the number of square inches in the area of the polygon shown? Express your answer as a mixed number. 24. _____



25. Let $\diamondsuit$ be defined as $\diamondsuit(a,b) = \sqrt{a^2+b^2}$, for all real numbers a and b. Find $\diamondsuit(\diamondsuit(1,2), \diamondsuit(3,4))$ and express in simplest radical form. 25. _____

26. The diagram consists of three nested squares. Find the ratio of the area of the smallest square to the area of the largest square. Express your answer as a common fraction. 26. _____



27. The coordinates of three vertices of a parallelogram are $(-3,1)$, $(2,5)$, and $(4,1)$. Find the sum of the coordinates of the fourth vertex which is in the third quadrant. 27. _____

28. What is the number of degrees in the acute angle formed by the hands of a clock at 6:44? 28. _____

29. The point $(0,8)$ lies on the graph of the curve defined by $y = c \cdot 2^x$. What is the value of y when $x = 2$? 29. _____

30. In chess, a knight moves in an L-shape manner—two spaces in one direction and one space in a direction perpendicular to the first direction—as shown. Beginning in and including the upper-left corner (marked *), what is the most number of squares in a 4×4 checkerboard that a knight can visit without visiting any square more than once? 30. _____