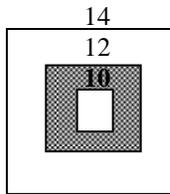


**MATHCOUNTS**  
**State Sprint Round**  
**1996-1997**

1. After spending one-half of her money and losing one-third of the remainder, Sally was left with just enough money to buy a magazine and a candy bar. The price of the magazine is 5 times the price of the candy bar, which is 75¢. How many dollars did Sally have initially? Express your answer as a decimal. 1. \_\_\_\_\_

2. What is the smallest number of 10" by 15" rectangular tiles required to form a square with no cutting or gaps? 2. \_\_\_\_\_

3. A target consists of three squares, measured in inches, as shown. A dart hits the target randomly. What is the probability that it hits the blackened region? Express your answer as a common fraction. 3. \_\_\_\_\_



4. Ioana has three ropes whose lengths are 39 inches, 52 inches, and 65 inches. She wants to cut the ropes into equal length pieces for magic tricks. No rope is to be wasted. What is the greatest number of inches possible in the length of each piece? 4. \_\_\_\_\_

5. Matt received change totaling \$1.85 in nickels, dimes, and quarters. He received at least one of each type. What was the least number he could have received? 5. \_\_\_\_\_

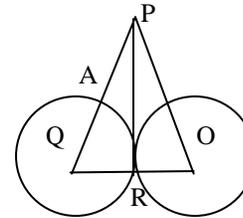
6. More than 20 and less than 50 people went to a concert. The price of each ticket was the same whole number of dollars, and the total cost was \$377. How many dollars did each individual ticket cost? 6. \_\_\_\_\_

7. Two cubes, each with six faces numbered 1, 2, 3, 4, 5, and 6 are tossed. Each cube lands with a prime number on top. What is the probability that the sum of these prime numbers is less than 9? Express your answer as a common fraction. 7. \_\_\_\_\_

8. In a sequence, each term after the second is the sum of the preceding two terms. The first and fifth terms are 3, what is the second term in the sequence? 8. \_\_\_\_\_

9. Each of the circles has a radius of 8 units.  $\overline{RP}$  is a common tangent to both circles.  $\overline{QA} = \frac{1}{6}\overline{QP}$ . 9. \_\_\_\_\_

How many units are in the perimeter of  $\triangle POQ$ ?



10. A car travels 40 kph for 20 kilometers, 50 kph for 25 kilometers, 60 kph for 45 kilometers, and 48 kph for 15 minutes. What was the average speed of the car, in kph? 10. \_\_\_\_\_

11. A grocer stacks apples in the shape of a square pyramid. The bottom layer is a 10 x 10 square, the top layer is one apple, and the  $n^{\text{th}}$  layer is an  $n \times n$  square. How many apples does she have in her pyramid? 11. \_\_\_\_\_

12. There are purple and green marbles in a bag in an unknown ratio, but when 15 green marbles are removed, the ratio becomes 2 purple marbles for every green marble. Then when 45 purple marbles are additionally removed, the ratio becomes 5 green marbles for each purple marble. How many green marbles were in the bag before any were removed? 12. \_\_\_\_\_

13. Twenty percent of the total cost of publishing the school newspaper during the academic year was the cost of the paper, while 80 percent of the cost was printing. Over the following summer, the cost of paper increased by twenty percent, while the cost of printing decreased by twenty percent. By what percent did the total cost of publishing the paper decrease? 13. \_\_\_\_\_

14. How many square units are in the area of a triangular region bounded by the x-axis, the y-axis, and the line with equation  $3x - y - 6 = 0$ ? 14. \_\_\_\_\_

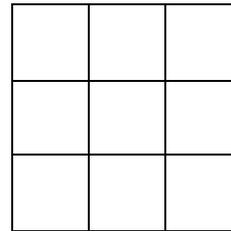
15. In the finite sequence 7, 8, 6, 9, 5, 10, ..., 0, the odd-numbered terms decrease by 1 and the even-numbered terms increase by 1. The last term of the sequence is zero. What is the sum of these terms? 15. \_\_\_\_\_

16. How many two-digit integers are increased exactly nine when the digits are reversed? 16. \_\_\_\_\_

17. An acute angle of a right triangle measures  $3x + 27$  degrees. Given that  $x$  is an integer, how many possible values are there for  $x$ ? 17. \_\_\_\_\_

18. Given that both  $p$  and  $p + 1$  are prime numbers, what is the least positive composite number that is not divisible by  $p$  nor  $p + 1$ ? 18. \_\_\_\_\_

19. The whole numbers 11 through 19 are arranged in the squares shown so that the sum of the numbers in each row, column, and diagonal is the same. What is the common sum? 19. \_\_\_\_\_



20. The number of students in Theresa's graduating class is more than 50 and fewer than 100 and is 1 less than a multiple of 3, 2 less than a multiple of 4, and 3 less than a multiple of 5. How many students are in Theresa's graduating class? 20. \_\_\_\_\_

21. What is the sum of the two least positive integers each of which has exactly eight positive factors? 21. \_\_\_\_\_

22. Claire jogs 6 miles per hour and Megan jogs 5 miles per hour. They start together at their campsite and jog to an outpost 18 miles away. When Claire gets there, she immediately turns around and heads back towards Megan. How many miles from the outpost will they be when they meet? Express your answer as a mixed number. 22. \_\_\_\_\_

23. The side length of a square is 16 cm long. The midpoints of each side are joined to form a second square. The process of joining the midpoints of the sides of the innermost square is repeated. What is the number of centimeters in the perimeter of the fifth square?

23. \_\_\_\_\_

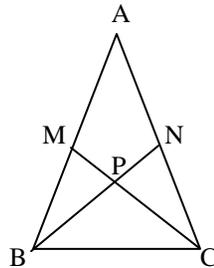
24. Each lateral edge of a hexagonal pyramid is 13 inches long. The altitude is 12 inches long. How many inches are in the perimeter of the hexagonal base?

24. \_\_\_\_\_

25. In equilateral  $\triangle ABC$ , points M and N are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively.

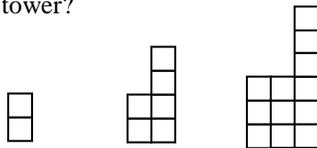
25. \_\_\_\_\_

Given that the area of  $\triangle BPC$  is 18 square units, how many square units are in the area of  $\triangle ABC$ ?



26. The first three towers in a sequence are shown. The  $n^{\text{th}}$  tower is formed by stacking  $n$  blocks on top of an  $n \times n$  square of blocks. How many blocks are in the 99<sup>th</sup> tower?

26. \_\_\_\_\_



27. How many three-digit numbers between 100 and 300 are there for which one of the digits equals the sum of the other two?

27. \_\_\_\_\_

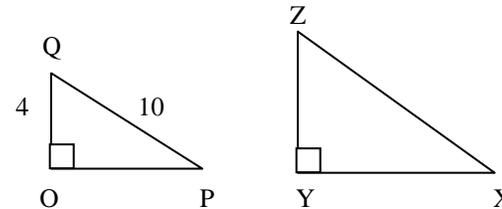
28. A train is formed exactly as shown by connecting regular hexagons and regular triangles, alternately, beginning with a hexagon. The side length of each polygon is 1 cm. How many hexagons are in a train that has a perimeter of 426 cm?

28. \_\_\_\_\_



29. Right  $\triangle POQ$  is similar to  $\triangle XYZ$ . Given that  $\overline{XY} = \sqrt{189}$  units, how many units are in the length of  $\overline{ZY}$ .

29. \_\_\_\_\_



30. Two integers are randomly selected from the set of integers greater than or equal to  $-5$  and less than or equal to  $5$ . The two numbers need not be different. What is the probability that the sum of the two integers is less than their product?

30. \_\_\_\_\_