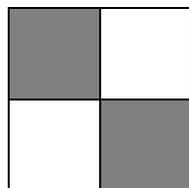
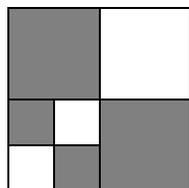


MATHCOUNTS
Team Round
1996-1997

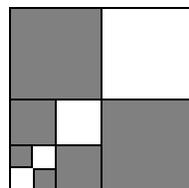
1. The sum of the reciprocals of the first four consecutive, positive integers is greater than two. What is the least number of consecutive, positive integers necessary to make the sum of the reciprocals greater than three? 1. _____
2. How many positive three digit numbers can be formed, such that the sum of the three digits is a prime number less than 10? 2. _____
3. Ms. Salman purchases a rectangular lot with an area of 1,200 sq. ft., which has a length equal to three times its width. She wishes to divide the lot into exactly 2 plots having equal area and install fencing along the perimeters of both lots. What is the least number of feet of fencing that she must buy? 3. _____
4. Randomly select a whole number such that $0 < x < 100$. What is the probability that x equals the sum of two perfect squares? Express your answer as a common fraction. 4. _____
5. At each stage, the square at the lower left is divided into four congruent square regions, 2 of which are shaded. The area of the entire square (including shaded and unshaded parts) is 256 square units. How many square units are in the shaded area at the fifth stage? Express your answer as a decimal. 5. _____



Stage 1



Stage 2



Stage 3

6. A rectangle is inscribed in a circle with a radius 5 units long. The ratio of the dimensions of the rectangle is .75. How many units are in the perimeter of the rectangle? 6. _____
7. A data set consists of 151 digits, used to write all of the natural numbers from 1 to 80 inclusive. What is the arithmetic mean of the digits in the data set? Express your answer as a decimal to the nearest tenth. 7. _____
8. Unit cubes are glued together to make a cube several units on each side. Some of the faces of this large cube are painted. When the cube is taken apart, there are exactly 45 unit cubes without any paint. How many unit cubes are used to make the larger cube? 8. _____
9. A robot is programmed to do the following moves in a coordinate system: start at the origin facing in the positive y direction and move one unit. For each subsequent move, it is to turn 90° clockwise and go forward 1 unit further than it went in the previous move. What is the sum of the coordinates of the robot's position after it just moved 14 units in one direction? 9. _____
10. Two circles are internally tangent. The smaller circle is also tangent to two perpendicular radii of the larger circle. What is the ratio of the area of the small circle to the area of the large circle? Express your answer as a decimal, rounded to the nearest hundredth? 10. _____