MATHCOUNTS
State Sprint Round
1998-1999

1. Calculate: $\frac{10!-8!}{10!+8!}$. Express your answer as a common fraction. 1. _____

2. How many pairs of vertical angles are formed by five distinct lines that have a common point of intersection? 2. _____

3. What is the unit's digit of $7^{21}-3^{84}$? 3. _____

4. Calculate: $2^{25}\times 5^{28}$. Express your answer in scientific notation. 4. _____

5. Two vertical poles are 16 feet apart. What is the minimum number of feet in the length of a rope that connects the top of the 50-foot pole to the top of the 20-foot pole? 5. _____

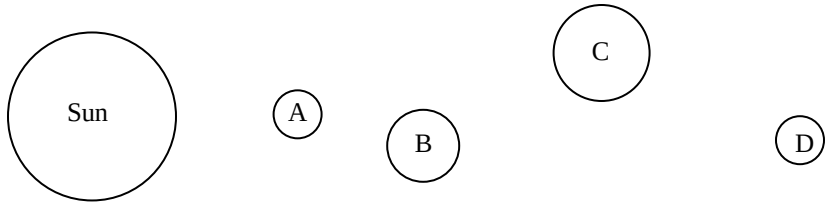
6. Calculate: $\sqrt[3]{4^5+4^5+4^5+4^5}$ 6. _____

7. How many different three-digit security codes digits 1-5, if the second digit cannot be the same as the first, and the third cannot be the same as the second? 7. _____

8. Mohan is selling cookies at the economics fair. As he decides how to package the cookies, he finds that when he bags them in groups of 4, he has 3 left over. When he bags them in groups of 5, he has 2 left over. When he bags them in groups of 7, he has 4 left over. What is the least number of cookies that Mohan could have? 8. _____

9. What is the sum of (37037•3) + (37037•6) + (37037•9)? 9. _____

10. A solar system contains four planets as shown. Planet A completes an orbit around the sun in 20 months; Planet B completes an orbit in 36 months; Planet C completes an orbit in 40 months; and Planet D completes an orbit in 48 months. If the planets lie along a straight line in May of the year 2000, what is the next year that they will again lie along the same line? Assume the orbits are circular and that a year is 12 months long. 10. _____



11. The cards in a deck of consecutive positive integers are equally spaced around a circle. If the card marked 5 is directly opposite the card marked 18, how many cards are in the deck? 11. _____

12. For what value of x does $\dfrac{8}{9} = \dfrac{x}{x + \dfrac{x}{x + x}}$? 12. _____

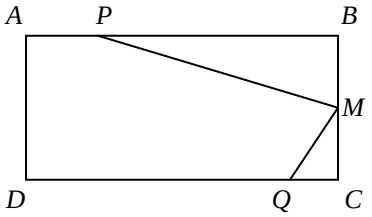
13. The volume of a rectangular box is increased by $66\frac{2}{3}\%$ when the length is increased by $33\frac{1}{3}\%$ and the height remains the same. By what percent did the width increase? 13. _____

14. A penny, a nickel, a dime, and a quarter are tossed simultaneously. What is the probability that at least two heads show and one of the heads showing is the dime? Express your answer as a common fraction. 14. _____

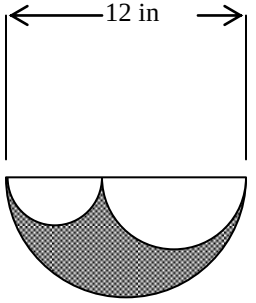
15. Seventy-two 50-cent pieces are lined side-by-side on a table. Joe replaces every second 50-cent piece with a quarter and leaves. Jane replaces every third remaining 50-cent piece with a dime and leaves. Jim replaces every fourth remaining 50-cent piece with a nickel and leaves. What is the number of dollars in the amount of money left on the table? Express your answer as a common fraction. 15. _____

16. In rectangle ABCD, points P and Q lie on $\overline{AB}$ and $\overline{DC}$ respectively. $\angle PMQ$ is a right angle, M is the midpoint of $\overline{BC}$, and $PB = \frac{4}{3}BC$. 16. _____

What is the ratio $PM:MQ$?
Express your answer as a common fraction.



17. The ratio of the radii of the unshaded semicircles is 1:2, and the unshaded semicircles are tangent to each other. What is the number of square inches in the area of the shaded region? 17. _____
Express your answer in terms of π.



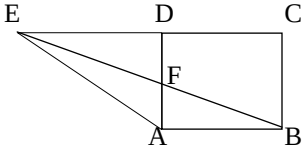
18. The house numbers of the seven houses on Avni's side of the street are consecutive even numbers. The sum of those numbers is 4522. Avni's house number is the median of the house numbers on her side of the street. What is Avni's house number? 18. _____

19. The line that contains the point (6,3) is perpendicular to the line whose equation is $3x + 2y = 6$. What is the sum of the coordinates of the y-intercept of this line? 19. _____

20. A gumball machine contains 9 red, 7 white, and 8 blue gum balls. What is the least number of gum balls a person must buy to ensure getting four balls of each color? 20. _____

21. When a positive integer n is multiplied by 7, the result is 3 less than Maria's favorite number. When $n + 2$ is multiplied by 5, the result is 1 less than Maria's favorite number. What is Maria's favorite number? 21. _____

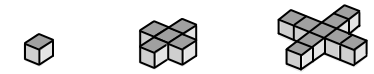
22. In the diagram, D is the midpoint of $\overline{EC}$, and the area of $\triangle$ EDF is 4 square centimeters. What is the number of square centimeters in the area of rectangle ABCD? 22. _____



23. An edge of a cube is 1 inch. A space diagonal of this cube is used as an edge to form a second cube. A space diagonal of the second cube is used as an edge to form a third cube. The process is repeated. How many inches are in the length of the fifth cube? 23. _____

24. What is the number of square units in the area of the region bounded by the graphs of $y = -|x| + 2$ and $y = |x| - 2$ 24. _____

25. The edge length of each cube shown is 1 cm. Four Cubes are added at each stage. What is the number of cubic centimeters in the volume of the figure in the 20^{th} stage? 25. _____



Stage 1 Stage 2 Stage 3

26. In June 1963, British tennis player Michael Sangster hit the fastest serve ever recorded, clocked at 154 mph. What was the speed of the ball in feet per second. Round your answer to the nearest whole number. 26. _____

27. When a number x is added to both the numerator and denominator of $\frac{1}{4}$, the result is a fraction <u>equivalent</u> to $\frac{2}{3}$. When a number y is added to both the numerator and denominator of $\frac{1}{5}$, the result is a fraction <u>equivalent</u> to $\frac{2}{3}$. What is the sum of $x + y$? 27. _____

28. After collecting for the candy drive, Sally had eight $1-bills, five $5-bills,and three $10-bills. She randomly selected three bills *without* replacement. What is the probability that she chose one of each type of bill? Express your answer as a common fraction.

28. _____

29. In a rectangle ABCD, H is the midpoint of $\overline{BC}$, E lies on $\overline{AD}$, and F lies on $\overline{AB}$. In rectangle CEFG, H lies on $\overline{FG}$ and HG = 3 in. $m\angle DEC = 45^\circ$. What is the number of square inches in the *positive* difference between the two rectangles?

29. _____

30. A two-lane track is formed by a rectangle and two semicircles, one semicircle at each end as shown. One lap around the track in the center of the shortest lane measures 400 meters. Each lane is 1.25 meters wide. How many more meters will a person who runs a lap in the center of the longer lane run than a person who runs a lap in the center of the shorter lane? Express your answer as a common fraction in terms of π.

30. _____