

MATHCOUNTS
Sprint Round
2000

1. What is the number of distinct ways of arranging the letters in the word AVERAGE? 1. _____

2. Joey's strategy for his first marathon (26.2 miles) was to run 2 miles, walk 1 mile, run two miles, walk one mile, and continue this pattern until he completed the race. Joey's average running pace is 8 minutes per mile, and his average walking pace is 16 minutes per mile. How many minutes will it take Joey to complete the marathon?
Express your answer as a decimal to the nearest tenth. 2. _____

3. Compute: $5 - 7(5^2 - 3^3)^4$ 3. _____

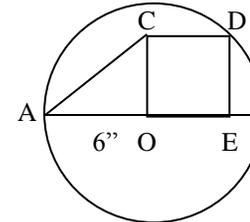
4. The chickens and pigs in Farmer McCoy's barn have a total of 50 heads and 170 legs. How many pigs are in the barn? 4. _____

5. Alan copied a picture to $66\frac{2}{3}\%$ of its original size and gave the copy to Beth. Beth then reduced her copy to $66\frac{2}{3}\%$ of its size and gave the copy to Carl. At what percent will Carl have to photocopy the picture to return it to its original size? 5. _____

6. The measures of the sides of an isosceles trapezoid are in the ratio of 3:4:3:6. The perimeter of the trapezoid is 48 cm. What is the number of square centimeters in the area of the trapezoid? Express your answer in simplest radical form? 6. _____

7. Compute: $\frac{4! + 3!}{3! + 2!}$. 7. _____
Express your answer as a decimal to the nearest hundredth.

8. In the diagram shown, COED is a square. The radius of circle O is 6 in. What is the number of inches in AC? 8. _____
Express your answer in simplest radical form.



9. A large red cube is dipped into red paint and then divided into 125 smaller congruent cubes. One of the smaller cubes is then randomly selected. What is the probability that the cube selected will have at least 25% of its surface painted red?
Express your answer as a common fraction. 9. _____

10. How many seconds longer is 2% of an hour than 30% of a minute? 10. _____

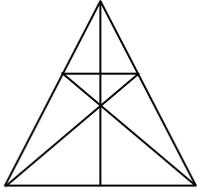
11. If $2^x \cdot 9^y$ is equal to the four-digit number $2x9y$ where x is the hundreds digit and y is the units digit, what is the product of x and y ? 11. _____

12. What is the units' digit of $(133^{13})^3$?

12. _____

13. How many triangles are in this figure?

13. _____

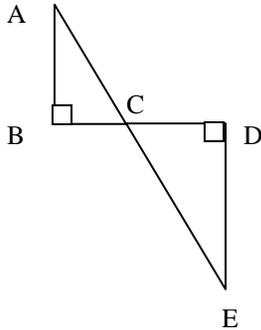


14. It is possible to earn 0, 1, 3, 7, or 10 points with each shot in the game of *Blippy*. How many positive scores less than 30 cannot be made in 3 shots?

14. _____

15. In the diagram $BD = 6$ km, $AB = 3$ km, and $DE = 5$ km. What is the number of kilometers in AE ?

15. _____



16. The coordinates of one of the endpoints of a diagonal of a rectangle are $(-4, 2)$, and the coordinates of the point of intersection of the diagonals are $(1, -1)$. The sides of the rectangle are parallel to the axes. What is the number of square units in the area of the rectangle?

16. _____

17. What is the value of the sum of:

$$2^{-1} + 2^{-2} + 2^{-3} + \dots + 2^{-9} + 2^{-10} ?$$

Express your answer as a common fraction.

17. _____

18. Given that $9a^2 - 8b^2 = 1800$ and $\frac{a}{b} = \frac{4}{3}$,

what is the product of ab ?

18. _____

19. There are 5 red, 7 white and 9 black cards in a stack. How many cards must be chosen to guarantee three of the same color?

19. _____

20. Start at the M in the diagram and form a path by moving to an adjacent letter to the right, left, up or down. How many paths spell the word MATH?

20. _____

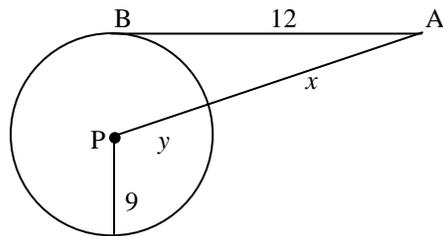
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21. Of the 400 eighth-graders at Pascal Middle School, 117 take algebra, 109 take advanced computer, 114 take industrial technology. Furthermore, 70 take **both** algebra and advanced computer, 34 take **both** algebra and industrial technology, and 29 take both advanced computer and industrial technology. Finally 164 students take none of these courses. How many students take all three courses?

21. _____

22. Given that \overline{AB} is tangent to circle P at B, what is the ratio $\frac{x}{x+y}$? Express your answer as a common fraction.



23. Melissa is driving a sports utility vehicle along the interstate at a constant speed of 55 mph. A sports car one-half mile behind her that is moving at a constant speed passes her in 60 seconds. How many miles per hour is the speed of the sports car?
24. How many ordered triples of three prime numbers exist for which the sum of the members of the triple is 24?
25. The lengths of the sides of an isosceles triangle ABC are $3x + 62$, $7x + 30$, and $5x + 50$ feet. What is the least possible number of feet in the perimeter of triangle ABC?
26. Janelle averages 40 kilometers per hour biking on level ground. She averages 60% of her level-ground speed riding uphill, and she averages 120% of her level-ground speed riding downhill. The course is level for 5 kilometers, uphill for 6 kilometers, and then downhill for 6 kilometers. How many kilometers per hour is her average speed for the entire course?

27. A bowl contains fewer than 50 marbles, and each is red, green or blue. The probability of drawing a red marble is $\frac{2}{5}$ and the probability of drawing a green marble is $\frac{3}{7}$. If two marbles are drawn without replacement, what is the probability that **both** are blue? Express your answer as a common fraction.

28. Brent likes to dilute his lemonade. He starts with a full cup and drinks $\frac{1}{3}$ of its contents. He then fills the cup with water, stirs the contents, and again drinks $\frac{1}{3}$ of its contents. He repeats the process until he has consumed one cup of liquid. What part of the original of lemonade remains in the cup? Express your answer as a common fraction.

29. The sum of the digits of a three-digit number is 26. The number is then multiplied by 7, then 11, and finally by 13. How many times will the digit 9 occur in the final product?

30. Joe bought a pumpkin that cost 10¢ more per pound than his sister's. Together, the two pumpkins weighed 20 pounds, but Joe's pumpkin was heavier. Joe paid \$3.52, and his sister paid \$.48. How many pounds did Joe's pumpkin weigh?