MATHCOUNTS
Team Round
2000

1. A train traveling at 60 miles per hour reaches a tunnel. The front of the train enters the tunnel, and 40 seconds later, the front of the engine exits the tunnel. How many feet are in the length of the train?

1._____

2. A lock has 5 buttons numbered 1-5. The lock is opened by pushing two buttons simultaneously and then pushing one button alone. How many combinations are possible?

2._____

3. During a game of *Scrabble*, Alexandria had tiles with the letters: R, O, O, V, G, N, and O. She rearranged the tiles to form all possible *distinct* 7-letter arrangements. In how many ways could she have arranged the tiles?

3._____

4. A circle is inscribed in a larger regular hexagon, and a smaller regular hexagon is inscribed in the circle. What is the ratio of the <u>area</u> of the <u>*smaller*</u> hexagon to the area of the <u>*larger*</u> hexagon? Express your answer as a common fraction.

4._____

5. An 11-member committee makes its decisions by simple majority vote: if 6 *or* more of the members vote in favor of an issue, the issue is passed. In how many ways can 6 or more members vote to pass an issue?

5._____

6. A pair of positive integers (x, y) satisfies the equation $31x + 29y = 1125$. What is $x + y$?

6._____

7. A triangle is formed by connecting the points $(-5, 0)$, $(0, 6)$, and $(5, 0)$. The resulting triangle is the rotated about the x-axis to form a solid. The original triangle is then rotated about the y-axis to form a different solid. What is the positive difference between the number of cubic units in the volumes of the two resulting solids? Express your answer in terms of π.

7._____

8. A group of 25 friends were discussing a large positive integer. "It can be divided by 1," said the first friend. "It can be divided by 2," said the second friend. "And by 3," said a third friend. "And by 4," added a fourth friend. This continued until everyone had made such a comment. If exactly two friends were incorrect, and those two friends said consecutive numbers, what was the least possible integer they were discussing?

8._____

9. A committee of three people is to be randomly selected from a group of three men and two women, and the chairperson will be randomly selected from the committee. What is the probability that the committee will have exactly two women and one man, and that the chairperson will be a woman?

9._____
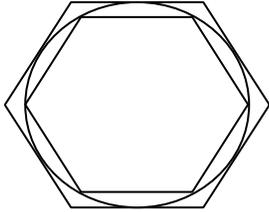
10. Big Ben, a famous 12-hour clock in London, England, sounds four notes on the quarter-hour, eight notes on the half-hour, twelve notes on the three-quarter hour, and sixteen notes on the hour. Furthermore, it strikes one note for each hour, on the hour; for example, it strikes five additional notes at 5 o'clock. How many notes will Big Ben strike in a twenty-four hour period?

10._____