1. A point (x, y) is randomly selected such that $0 \leq x \leq 8$ and $0 \leq y \leq 4$. What is the probability that both $x \leq 2$ and $y \leq 2$? Express your answer as a common fraction.

1. _____

2. The integers $1, 2, 3, \ldots, n$, are equally-spaced around a circle. The numbers 17 and 54 are located at opposite ends of the same diameter. What is n?

2. _____

3. Three circles are externally tangent to one another. The radius of each of the circles is 2 units. A belt fits tightly around the three circles. Find the number of units in the length of the belt. Express your answer in terms of π.

3. _____
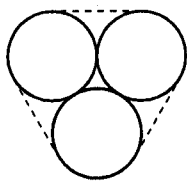
4. Both dimensions of a photograph are decreased by 50% to create a wallet-sized picture, and both dimensions of the original photograph are increased by 50% to create a portrait-sized picture. Find the ratio of the area of the wallet-sized picture to the area of the portrait-sized picture. Express your answer as a common fraction.

4. _____

wallet

original

portrait
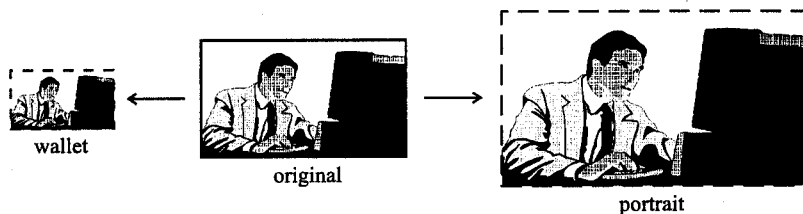
5. A *relatively prime date* is a date for which the number of the month and the number of the day are relatively prime. For example, June 17 is a relatively prime date because the greatest common factor of 6 and 17 is 1. How many relatively prime dates are in the month with the fewest relatively prime dates?

5. _____

6. The pattern below continues by adding an outer layer of hexagons at each stage. How many hexagons will be in stage 10?

6. _____

Stage 1   Stage 2   Stage 3
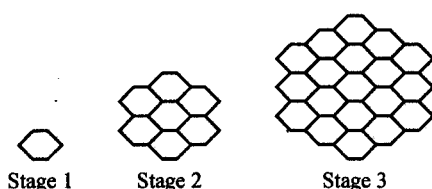
7. A sponsor agrees to donate \$5.00 plus an additional 5¢ for each problem Aaron solves. Another sponsor agrees to donate \$2.00 plus an additional 15¢ for each problem Aaron solves. How many problems must Aaron solve so that both sponsors will donate the same amount?

7. _____

8. The perimeter of an isosceles triangle is 40 centimeters. The congruent side lengths are integers. What is the maximum number of centimeters in the length of the third side?

8. _____

9. Let f and g be functions as defined below. What is $g(f(1))$?

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0; \\ 4 - x & \text{if } x \geq 0. \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2x & \text{if } x < 5; \\ 3x & \text{if } x \geq 5. \end{cases}$$

9. _____

10. ABCD is a square. How many squares have two or more vertices in the set $\{A, B, C, D\}$?

10. _____