STRETCHES

The Stretches, created to give Mathletes practice with specific subject areas, focus on the following areas:

Geometry	Topics in Plane and Solid Geometry
Algebra	Symbolic Manipulation and Algebraic Thinking
Factoring	Probability and Number Sense Involving Factors

As part of the MATHCOUNTS coaching phase, the Stretches can be used to prepare Mathletes for more advanced problem-solving situations. They can be used prior to the Warm-Ups and Workouts to introduce mathematical topics, or they can be used to teach and reinforce concepts after Mathletes have attempted the Warm-Ups and Workouts. Finally, they can be used when preparing for competition to aid Mathletes with troublesome concepts.

Answers to the Stretches include one-letter codes, in parentheses, indicating appropriate problem-solving strategies. However, students should be encouraged to find alternative methods of solving the problems; their methods may be better than the one provided! The following strategies are used: **C** (Compute), **F** (Formula), **M** (Model/Diagram), **T** (Table/Chart/List), **G** (Guess & Check), **S** (Simpler Case), **E** (Eliminate) and **P** (Patterns).

MATHCOUNTS Symbols and Notation

Standard abbreviations have been used for units of measure. Complete words or symbols are also acceptable. Square units or cube units may be expressed as units^2 or units^3.

Geometry Stretch

1. _____   How many square units are in the greatest area that can be enclosed by a rectangle whose perimeter is 20 units?

2. _____   What is the number of units in the perimeter of a triangle bound by the x-axis, the y-axis and the line $y = -\dfrac{3}{4}x + 3$?

3. _____   Circle E is inscribed in square ABCD. If the length of segment AB is 4 inches, how many square inches are in the area of the shaded region? Express your answer in terms of π .
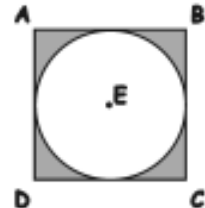
4. _____   Three angles of a pentagon have measures 88°, 124° and 92°. If the measures of the remaining 2 angles are equal, what is the measure, in degrees, of one of the remaining angles?

5. _____   What is the number of inches in the height of an equilateral triangle whose perimeter is 30 inches? Express your answer in simplest radical form.

6. _____ $\triangle ABC$ and $\triangle DEF$ are similar right triangles. The two legs of $\triangle ABC$ are 5 cm and 6 cm in length. If the area of $\triangle DEF$ is 135 cm², what is the number of centimeters in the length of the longer leg of $\triangle DEF$?

7. _____   What is the number of square units in the area of the regular hexagon ABCDEF if segment DE is equal to 4 units? Express your answer in simplest radical form.

8. _____   Segment AB has endpoints at A (-1, 2) and B (3, 1). Segment AB is reflected over the y-axis such that A becomes A' and B becomes B'. What is the positive difference between the lengths of segment AA' and segment BB'?

9. _____   If the length of the edge of a cube is increased by 50%, what is the percent increase in the volume of the cube? Express your answer to the nearest whole number.

10. _____   A flag pole is placed in the sand with the top of the flag pole standing 10 feet above the ground. A 26-foot string is attached to the top of the flag pole at point A. Holding the string to the ground, what is the number of square feet in the area of the largest circle that can be drawn in the sand with the end of the string? Express your answer in terms of π .
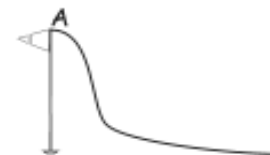
Algebra Stretch

1. _____ A straight line passes through the three points (3, -4), (5, 1) and (7, y). What is the value of y ?

2. _____ What is the value of $\dfrac{(x-1)!}{(x-3)!}$ for x = 100?

3. _____ If I = $^E/_R$, E = 87 and I = 3, then what is the value of R?

4. _____ What is the value of *n* if $\dfrac{1}{4} = \dfrac{1}{5} + \dfrac{1}{n}$?

5. _____ The sum of 7 consecutive integers is 413. What is their mean?

6. _____ The sum of two numbers is 5 and their difference is 11. What is the product of the two numbers?

7. _____ Point P is the point of intersection of the horizontal line through (4, 2) and the vertical line through (-5, 5). What is the sum of the coordinates of point P?

8. _____ There are only bicycles and tricycles in Tracy's backyard. She correctly counted a total of 30 seats and 70 wheels in the backyard. How many tricycles are in her backyard?

9. _____ Carrie has an 88% average in biology after all four of the marking periods. If the final exam counts twice as much as each of the four marking periods, what percent must Carrie make on the final exam to have a final average of 90% for the course?

10. _____ The temperature *t* of the air in degrees Fahrenheit is related to the number *n* of chirps a cricket makes in a minute by the formula: $t = \dfrac{n}{4} + 40$. How many times per minute does a cricket chirp when the air temperature is 52° F?

Factoring Stretch

1. _____   What is the sum of all of the distinct, positive prime factors of 1260?

2. _____   What is the product of all of the values of n that make $546{,}324{,}16n$ divisible by 6?

3. _____   Billy tosses one fair 6-sided die with faces labeled 1 through 6. He records the outcome. Billy does this three more times and the product of his four outcomes is 120. How many possible combinations of 4 rolls could he have rolled? (Rolling a 1, 1, 2, 2 is considered the same combination as rolling a 1, 2, 2, 1.)

4. _____   The numbers 1 - 400, inclusive, are put into a hat. What is the probability that the first number chosen at random is a multiple of 4 or 17? Express your answer as a common fraction.

5. _____   If a and b are distinct, odd primes, then how many distinct positive factors does $4a^2b^3$ have?

6. _____   What is the smallest positive integer that has 2, 3, 4, 6, 7 and 12 as factors?

7. _____   What is the sum of the three greatest consecutive integers less than 200 for which the least number has 4 as a factor, the second number has 5 as a factor and the greatest number has 6 as a factor?

8. _____   What is the greatest whole number less than 150 that has an odd number of distinct positive factors?

9. _____   Find n such that $2! \cdot 3! \cdot 4! \cdot n = 8!$.

10. _____   What is the smallest positive integer n for which 72 is a factor of n! ?

Geometry Stretch

Answers

1.	25	(G, P)	5.	$5\sqrt{3}$	(F)	8.	4	(M, P)
2.	12	(F, M)	6.	18	(T, S)	9.	238	(S, F)
3.	$16 - 4\pi$	(F)	7.	$24\sqrt{3}$	(F)	10.	$576\,\pi$	(M, F)
4.	118	(G, E, F)						

Algebra Stretch

Answers

1.	6	(G, M, E)	5.	59	(C, P, G)	8.	10	(F, G, E)
2.	9702	(P)	6.	-24	(F, G)	9.	94	(F, G, E)
3.	29	(C)	7.	-3	(M)	10.	48	(P, F)
4.	20	(C, G)						

Factoring Stretch

Answers

1.	17	(C)	5.	36	(F, P)	8.	144	(P, E)
2.	16	(F, E, G)	6.	84	(C)	9.	140	(C, E, P)
3.	3	(P, G, E)	7.	555	(E)	10.	6	(P, E)
4.	$\dfrac{59}{200}$	(P, E)						