

## 2002-2003 MATHCOUNTS School Handbook

### **STRETCHES**

The Stretches, created to give Mathletes practice with specific subject areas, focus on the following areas:

<b>Proportional Reasoning</b>	Rates, Ratios and Fractions
<b>3-D Geometry</b>	Topics in Solid Geometry
<b>Counting Combinatorics</b>	Counting Techniques and Problems Involving Combinatorics


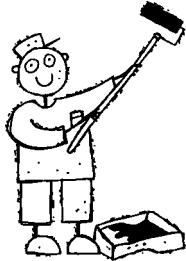
As part of the MATHCOUNTS coaching phase, the Stretches can be used to prepare Mathletes for more advanced problem-solving situations. They can be used prior to the Warm-Ups and Workouts to introduce mathematical topics, or they can be used to teach and reinforce concepts after Mathletes have attempted the Warm-Ups and Workouts. Finally, they can be used when preparing for competition to aid Mathletes with troublesome concepts.

Answers to the Stretches include one-letter codes, in parentheses, indicating appropriate problem-solving strategies. However, students should be encouraged to find alternative methods of solving the problems; their methods may be better than the one provided! The following strategies are used: **C** (Compute), **F** (Formula), **M** (Model/Diagram), **T** (Table/Chart/List), **G** (Guess & Check), **S** (Simpler Case), **E** (Eliminate) and **P** (Patterns).

#### **MATHCOUNTS Symbols and Notation**

Standard abbreviations have been used for units of measure. Complete words or symbols are also acceptable. Square units or cube units may be expressed as  $\text{units}^2$  or  $\text{units}^3$ .

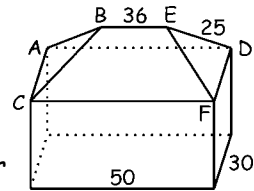
# Proportional Reasoning Stretch

- \_\_\_\_\_ A dozen cookies cost \$3.99. At the same cost per cookie, how many dollars should eight cookies cost? Express your answer to the nearest hundredth.
- \_\_\_\_\_ Marian traveled at an average rate of 65 miles per hour for 15 hours. Kristen travels at an average rate of 50 miles per hour. How many hours does it take Kristen to travel the same distance Marian traveled? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ A recipe requires three cups of flour and two eggs to make eight servings of a cake. How many cups of flour are needed to make 20 servings of the same cake? Express your answer as a decimal to the nearest tenth. 
- \_\_\_\_\_ To receive an attendance bonus of 20 points in her history class, Phyllis must attend at least 95% of the 180 history classes during the school year. By the end of January, she has missed six classes. What is the maximum number of history classes she can miss the rest of the school year and still get the attendance bonus?
- \_\_\_\_\_ Thirty percent of a number is 8.4. What is 37.5% of the number, as a decimal to the nearest tenth?
- \_\_\_\_\_ A price is reduced by 10% and then increased by 10%. What percent of the original price is the new price?
- \_\_\_\_\_ How many meters per second faster is 30 meters per second than 100 kilometers per hour? Express your answer as a decimal to the nearest tenth.
- \_\_\_\_\_ The ratio length:width:height of a box is 3:1:2. If the height of the box is 10 cm, how many square centimeters is the surface area of the box?
- \_\_\_\_\_ It takes four painters working at the same rate 1.25 work-days to finish a job. If only three painters are available, how many work-days will it take them to finish the job, working at the same rate? Express your answer as a mixed number. 
- \_\_\_\_\_ A surveillance camera photographs a shoplifter in front of a 3.5-foot counter. In the photograph, the shoplifter appears 6 inches tall and the counter appears 4 inches high. How many inches tall is the shoplifter?

# 3-D Geometry Stretch

- \_\_\_\_\_ A cube with volume 8 cubic units is cut in half by a plane which contains two edges of the cube. What is the area, in square units, of the rectangle formed by the intersection of the plane and cube? Express your answer in simplest radical form.
- \_\_\_\_\_ A *space diagonal* of a polyhedron is a segment connecting two vertices of the polyhedron which are not contained in the same face of the polyhedron. How many space diagonals does a pentagonal prism have?
- \_\_\_\_\_ There are 12 diagonals that can be drawn on the faces of a cube. Two different diagonals are selected at random. What is the probability that the diagonals selected share a common point? Express your answer as a common fraction.

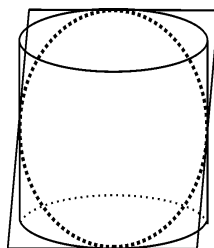
- \_\_\_\_\_ The dimensions, in feet, of a warehouse are given in the diagram. The base and the four walls are rectangles. The roof consists of two isosceles triangles ( $AB=BC=DE=EF$ ) and two isosceles trapezoids. A roofing company charges \$1.80 per square foot of roof to put a new roof on the warehouse. What will it cost, in dollars, to put a new roof on the warehouse? Express your answer to the nearest hundred dollars.



- \_\_\_\_\_ A grocer stacked oranges in a pyramid so that there were a square number of oranges in each layer (1 by 1 on top, then 2 by 2, then 3 by 3, etc.). If there are between 1500 and 2000 oranges in the stack, how many layers tall is the pyramid?
- \_\_\_\_\_ Shawn forms a ball of clay into a solid sphere of radius 6 cm. He then shapes it into a solid cube. What is the ratio of the surface area of the cube to the surface area of the sphere? Express your answer as a decimal to the nearest hundredth.
- \_\_\_\_\_ Marty has a  $3 \times 3 \times 3$  cube of unit cubes. If Marty then removes six of the unit cubes from the original cube, what is the greatest possible number of square units in the surface area of the resulting solid?
- \_\_\_\_\_ Let  $A(0, 0)$ ,  $B(18, 0)$  and  $C(10, 6)$  be the vertices of a triangular region that is rotated about the  $x$ -axis. What is the volume, in cubic units, of the solid that is formed? Express your answer in terms of  $\pi$ .
- \_\_\_\_\_ The four graphs on the right give the area of a horizontal slice of the figures on the left, as a function of the height from the bottom of the figure. Match each of the four figures with its corresponding graph.



- \_\_\_\_\_ The area of the ellipse, shown by the heavy dotted curve, is given by  $A = \pi ab$  where  $a$  and  $b$  are half the lengths of the axes of the ellipse. The ellipse was formed by cutting a right circular cylinder with radius 2 cm and height 3 cm by a plane, as shown. What is the number of square centimeters in the area of the ellipse? Express your answer in terms of  $\pi$ .



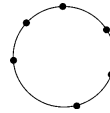
# Counting/Combinatorics Stretch

1. \_\_\_\_\_ Bob has stencils to paint the digits 2, 5 and 8. How many distinct three-digit house numbers can he paint, using only the stencils?

2. \_\_\_\_\_ At the end of a game, each of the five members of a basketball team shake hands with each of the five members of the other team, and all of the players shake hands with the two referees. How many handshakes occur?

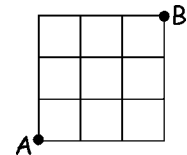


3. \_\_\_\_\_ Six points are drawn on a circle. How many distinct convex pentagons can be drawn using only these points as vertices?



4. \_\_\_\_\_ A nursery employee wishes to plant six Golden Delicious apple trees and two Bartlett pear trees in one row. How many distinct arrangements are possible?

5. \_\_\_\_\_ Each small square has sides of length 1 unit. How many distinct paths of length six units are there from A to B?



6. \_\_\_\_\_ A state with ten million cars plans to issue license plates which consist of any four letters followed by an  $n$ -digit number. If the state wants to have enough distinct license plates for all of the cars, what is the minimum possible value for  $n$ ?

7. \_\_\_\_\_ There are six tags numbered 1, 2, 2, 3, 3 and 4. Using these tags, how many distinct three-digit numbers can be formed such that two of the digits are the same?

8. \_\_\_\_\_ A teacher has made ten statements for a True-False test. Four statements are true and six are false. How many distinct answer keys could there be for the test?

9. \_\_\_\_\_ How many perfect square factors does the number 46,656 have?

10. \_\_\_\_\_ There are eight boys and six girls who are members of the trumpet section in the band. From the members of the trumpet section, a quintet is being formed. If the quintet must consist of three boys and two girls, how many quintets are possible?



# Proportional Reasoning Stretch

## Answers

- |    |      |        |    |      |        |     |                |           |
|----|------|--------|----|------|--------|-----|----------------|-----------|
| 1. | 2.66 | (C)    | 5. | 10.5 | (C)    | 8.  | 550            | (F, C, M) |
| 2. | 19.5 | (F, C) | 6. | 99   | (S, C) | 9.  | $1\frac{2}{3}$ | (F, C)    |
| 3. | 7.5  | (C)    | 7. | 2.2  | (C)    | 10. | 63             | (C, M)    |
| 4. | 3    | (C)    |    |      |        |     |                |           |

# 3-D Geometry Stretch

## Answers

- |    |                |           |    |      |              |     |                       |           |
|----|----------------|-----------|----|------|--------------|-----|-----------------------|-----------|
| 1. | $4\sqrt{2}$    | (M, F, C) | 5. | 17   | (F, G, C, T) | 8.  | $216\pi$              | (M, F)    |
| 2. | 10             | (M)       | 6. | 1.24 | (F)          | 9.  | 1-C, 2-B, 3-A, 4-D(E) |           |
| 3. | $\frac{5}{11}$ | (M, P)    | 7. | 78   | (M, C, T, S) | 10. | $5\pi$                | (F, C, M) |
| 4. | 4800           | (C, F)    |    |      |              |     |                       |           |

# Counting/Combinatorics Stretch

## Answers

- |    |    |                 |    |    |              |     |     |              |
|----|----|-----------------|----|----|--------------|-----|-----|--------------|
| 1. | 27 | (C, T, F)       | 5. | 20 | (F, P, M)    | 8.  | 210 | (F, C, M)    |
| 2. | 45 | (F, C, T, M)    | 6. | 2  | (C, T, F, G) | 9.  | 16  | (S, C, E, T) |
| 3. | 6  | (M, S, P, F, C) | 7. | 18 | (T, S, C, E) | 10. | 840 | (F, C, M)    |
| 4. | 28 | (P, F, C, M)    |    |    |              |     |     |              |