MATHCOUNTS

2004 Chapter Competition Solutions

Are you wondering how we could have possibly thought that a Mathlete® would be able to answer a particular Sprint Round problem without a calculator?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Target Round problem in less 3 minutes?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Team Round problem with less that 10 sheets of scratch paper?

The following pages provide solutions to the Sprint, Target and Team Rounds of the 2004 MATHCOUNTS® Chapter Competition. Though these solutions provide creative and concise ways of solving the problems from the competition, there are certainly numerous other solutions that also lead to the correct answer, and may even be more creative or more concise! We encourage you to find numerous solutions and representations for these MATHCOUNTS problems.

Special thanks to volunteer author Mady Bauer for sharing these solutions with us and the rest of the MATHCOUNTS community!

2004 CHAPTER COMPETITION

SPRINT ROUND QUESTIONS

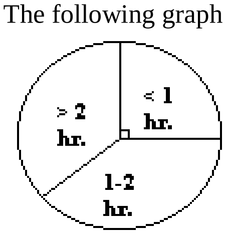
1. Each of the 3 math classes has 24 students. Each student is in only one of the math classes. Therefore, there are $3 \times 24 = 72$ students in the math classes. 72 **Ans.**

2. A triangle has sides of length 14 cm, 8 cm and 9 cm. The perimeter is the sum of the three sides. 14 + 8 + 9 = 31 **Ans.**

3. $x^2 + y^2 = 73$
Let's do this the easy way. Enumerate the squares less than 73:
1, 4, 9, 16, 25, 36, 49, 64
If x^2 were 1, then y^2 would be 72 but 72 isn't a square.
If x^2 were 4 then y^2 would be 69. Again, no luck.
If x^2 were 9 then y^2 would be 64. That works!
Therefore, x = 3 and y = 8.
3 + 8 = 11 **Ans.**

4. A *clever integer* is an even integer that is greater than 20, less than 120, and the sum of its digits is 9. Let's list the *clever integers*.
36, 54, 72, 90, 108
There are 5 *clever integers*.
Of these 54 and 108 are divisible by 27. (Note that 99 wasn't in there because it violates the sum of the digits. But if we had put it in the sequence, every third number starting with the first would be divisible by 27 because each of these numbers is 9 greater than the previous.)
The fraction of all *clever integers* that are divisible by 27 is 2/5. **Ans.**

5. The following graph shows TV viewing.



The graph shows that $\frac{1}{4}$ of the viewers watched less than 1 hour. (This is because there is a right angle or 90° which is $\frac{1}{4}$ of the 360° that makes up a circle.) Therefore, the percentage is 25%. The number of viewers who watched one hour or more is: 100 - 25 = 75% **Ans.**

6. $\frac{1}{2} - \frac{1}{3} = \frac{1}{x}$
$\frac{3}{6} - \frac{2}{6} = \frac{1}{x}$
$\frac{1}{6} = \frac{1}{x}$
Therefore, x = 6. **Ans.**

7. The points in this table lie on a straight line.

x	y
1	7
3	13
5	19

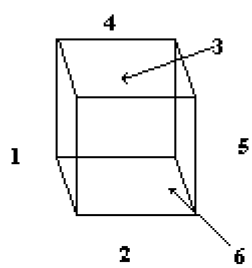
From this information we can figure out the equation for the line.
y = mx + b
7 = m(1) + b (Eq. 1)
13 = 3m + b (Eq. 2)
6 = 2m (Eq. 2 - Eq. 1)
3 = m
y = 3x + b
7 = 3(1) + b (substituting 1 for x and 7 for y)
Therefore, the equation for the line is:
y = 3x + 4
(28, t) is a point on that line. Therefore:
$t = 3 \times 28 + 4 = 84 + 4 = 88$ **Ans.**

8. (_6_,3_1) - (76,_35) = (_9,28_)
We must place a digit from 0 through 9 in each blank so the equation below is true. Well...this is fun... Hmmm... Let's change this into an addition problem that we know is true: (_9,28_) + (76,_35) = (_6_,3_1). Notice that for the one's column, only 6 + 5 will give us 11. So we have (_9,286) + (76,_35) = (_6_,3_1). We also now have a 1 that's been carried and will be included with the sum of the 8 and 3 in the ten's place, to give us a 2 in the ten's place of the answer: (_9,286) + (76,_35) = (_6_,321). For the hundred's place we know that we have a 1 that's been carried along with a missing number and a 2 that must get us to a sum of 3 or 13, so the missing number is 0, and we

have (_9,286) + (76,035) = (_6_,321). In the thousand's place, the 9 and 6 will give us a 5 in the answer, and will result in us placing the 5 and carrying a 1: (_9,286) + (76,035) = (_65,321). For the ten-thousand's place, we have the 1 that was carried along with another missing number and the 7 that must get us the 6 or 16 in the answer. This missing number must be an 8 since 1 + 8 + 7 = 16: (89,286) + (76,035) = (_65,321). And since this did result in 16, we also know that the final missing spot is a carried 1: (89,286) + (76,035) = (165,321). Now we have to add up the six digits we placed in the blanks: 8 + 6 + 0 + 1 + 5 + 2 = 22
Now that was fun, wasn't it? 22 **Ans.**

9. A pizza costs \$17.60 and it is cut into 16 equal pieces. Brad ate 6 or $\frac{6}{16} = \frac{3}{8}$ of the pizza. $\frac{3}{8} \times 17.60 = \$6.60$ **Ans.**

10. When the cube is created the following numbers are opposite each other:



For this orientation:
1 and 5 are the sides
3 and 6 are the back and front
4 and 2 are the top and bottom
(Hey, try drawing this yourself. It's not that easy!!!)
So there are 3 sets of numbers that are on the 4 lateral faces depending on which numbers are the top and bottom.
If 2 and 4 are the top and bottom, then the lateral faces are 1, 3, 5 and 6.
$1 \times 3 \times 5 \times 6 = 90$
If 1 and 5 are the top and bottom, then 2, 6, 3 and 4 are the lateral faces.
$2 \times 6 \times 3 \times 4 = 144$
Finally, if 3 and 6 are the top and bottom, then 1, 5, 2 and 4 are the lateral faces.
$1 \times 5 \times 2 \times 4 = 40$
The greatest possible lateral product for the cube is 144. **Ans.**

11. A cup contains \$1.25 in coins. There is at least one penny, one nickel and one dime in the cup. That is a total of:
\$.01 + \$.05 + \$.10 = \$.16
That means the other coins make up:
\$1.25 - \$.16 = \$1.09
The maximum number of nickels occurs by turning as much of \$1.09 into nickels as possible. There are \$1.09 ÷ \$.05 = 21 additional nickels that could be in the cup. This means the maximum number of nickels is 21 + 1 = 22. The minimum number of nickels is 1 because we can just use pennies to make up the \$1.09 if we want. There's nothing in the problem that says we can't.
22 + 1 = 23 **Ans.**

12. A snowboard that was priced at \$100 was discounted 50%. So the new price was: 100% - 50% = 50% of the old price. Then the price was reduced by 30% making the final price 100% - 30% = 70% of the intermediate price. 70% of 50% is 35%.
35% of \$100 = \$35
Of course you could still do this the normal way. 50% of \$100 is \$50. 30% of 50 is \$15. 50 - 15 = \$35
There is more than one way to figure out the price of a snowboard! \$35 **Ans.**

13. x + y = 153
$\frac{x}{y} = 0.7$
We can set the ratio of *x* and *y* equal to a ratio, rather than a decimal, and then see that multiplying the numerator and denominator by 10 creates numbers that are slightly too large, but multiplying each by 9 results in numbers in the ratio that do give us our desired sum of 153.
$$\frac{x}{y} = \frac{7}{10} = \frac{70}{100} = \frac{63}{90}$$
Therefore, $y - x = 90 - 63 = 27$ **Ans.**

14. We are given the set {3, 6, 12}. How many unique values can be created by forming fractions $\frac{x}{y}$ where x and y are distinct members of the given set? Since there are only three members in the set there will be 6 combinations. Let's just write them down.

$\frac{3}{6}, \frac{3}{12}, \frac{6}{3}, \frac{6}{12}, \frac{12}{3}, \frac{12}{6}$ Reduce the fractions.

$\frac{1}{2}, \frac{1}{4}, 2, \frac{1}{2}, 4, 2$

2 and $\frac{1}{2}$ are repeated twice so there are

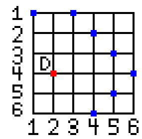
6 -2 = 4 unique values. 4 **Ans.**

15. There are 8 furlongs in a mile.
There are two weeks in a fortnight, or 14 days.
2800 furlongs were traveled during the fortnight or $\frac{2800}{14}$ = 200 furlongs per day.
$\frac{200\ furlongs/day}{8\ furlongs/mile}$ = 25 miles per day **Ans.**

16. How many integer palindromes are between 100 and 500?
Look at the values between 100 and 199. The numbers must start and end with 1 so the only value that can vary is the tens column, i.e., 101, 111, 121, 131, ..., 191 for a total of 10 palindromes. Similarly, there are 10 palindromes for each of 200-299, 300-399, and 400-499. 500 isn't a palindrome.
$10 \times 4 = 40$ **Ans.**

17. How many diagonals does a regular seven-sided polygon contain? I never remember the formula for this so I usually derive it again. The seven-sided polygon contains 7 vertices. Each vertex can create a diagonal with any vertex that is not itself or adjacent to itself. This means that 3 vertices cannot be used, leaving 7 - 3 or 4 vertices that any vertex can connect to in order to create a diagonal. There are 7 vertices, each with 4 diagonals, or 28 diagonals. But remember, a diagonal can go from point A to point B and that is the same as the diagonal that goes from point B to point A! We must divide by 2. $\frac{28}{2} = 14$
And the formula is $\frac{n \times (n-3)}{2}$ where n is the number of vertices in the polygon.
14 **Ans.**

18. We have a grid with point D.



Using the numbering system, D is at (2,4). We have to find all intersections that are exactly four blocks away from point D. Start by going 4 blocks to the right. This gets you to point (6, 4). Now start at D again and go 3 blocks to the right. To get four blocks away, you can go one block up or one block down. These are points (5, 3) and (5,5). Now start at D again and go 2 blocks to the right. You can go two blocks up or down to points (4, 2) and (4, 6). (Now suppose you said, wait a minute, I can go two blocks to the right, one up and then one to the right. That's (5, 3) and we already have it. But then you said, well I can go two blocks to the right, one up and one to the left. That would get you to (3,3) but this point really isn't 4 blocks away -- it's only 2 blocks away. Watch out for that!) So, continuing, start at D again and go one block to the right. Then you can go 3 up but you can't go 3 down. So you get only one point, (3, 1). Now, try going up. You can go up 3 and then must go either right one or left one. We've already done (3, 1) but (1,1) is new. Try going any other way and you'll either end up on a point you counted or you'll end up on a point that really isn't 4 blocks away. Counting up the blue dots we have 7 points.
7 **Ans.**

19. $a \lambda b = a^2 - b$
$3 \lambda 5 = 3^2 - 5 = 9 - 5 = 4$
$3^{3\lambda 5} = 3^4 = 81$
$4 \lambda 13 = 4^2 - 13 = 16 - 13 = 3$
$2^{4\lambda 13} = 2^3 = 8$
$(2^{4\lambda 13}) \lambda (3^{3\lambda 5}) = 8 \lambda 81 =$
$8^2 - 81 = 64 - 81 = -17$ **Ans.**

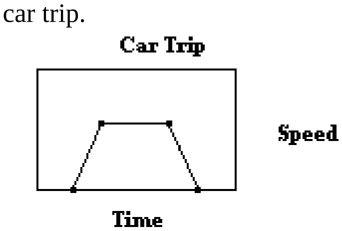
20. The Warloe sequence is:
... , 1, 2, 3, 6, 11, ...
where 2 is the 8th term in the sequence. Each term after the third term is the sum of the previous 3 terms. We first have to find the 13th term. Since 2 is the 8th term, 11 is the 11th term.
The 12th term is 3 + 6 + 11 = 20.
The 13th term is 6 + 11 + 20 = 37
The 7th term is 1.

The 6th term added to 1 + 2 must be the 9th term which is 3. 1 + 2 is 3 so the 6th term is 0.
..., 0, 1, 2, ...
The 7th term is 1 and the 6th term is 0. Their sum is 1 and the 8th term is 2. Therefore, 1 must be added to 1 to make 2 so the 5th term is 1.
..., 1, 0, 1, ...
The 6th term is 0 and the 5th term is 1. Their sum is 1 and the 7th term is 1. Therefore, 0 must be added to 1 to make 1, so the 4th term is 0.
..., 0, 1, 0, ...
The 5th term is 1 and the 4th term is 0. Their sum is 1 and the 6th term is 0. Therefore, -1 must be added to 1 to make 0, so the 3rd term is -1.
..., -1, 0, 1, ...
The 4th term is 0 and the 3rd term is -1. Their sum is -1 and the 5th term is 1. Therefore, 2 must be added to -1 to make 1, so the 2nd term is 2.
..., 2, -1, 0, ...
The 3rd term is -1 and the 2nd term is 2. Their sum is 1 and the 4th term is 0. Therefore, -1 must be added to 1 to make 0, so the first term is -1.
-1 + 37 = 36 **Ans.**
Wasn't that fun?

21. | -2a + 1 | < 13
Suppose (-2a + 1) is positive. Then:
-2a + 1 < 13
-2a < 12
2a > -12
a > -6
Now, suppose (-2a + 1) is negative.
-2a + 1 > -13
-2a > -14
a < 7
Therefore the range is -6 < a < 7.
If we sum up -5 through 6, all values will cancel themselves out (i.e., -5 + 5, -4 + 4, etc.) except the 6. 6 **Ans.**

22. A value, p, is randomly chosen from the integers 0 through 13 to make $\frac{p}{13}$. This is the coordinate of point A. The coordinate of point B is $\frac{2}{3}$.
Point A will range from 0 to 1. We need to find how many values of $\frac{p}{13}$ are within $\frac{1}{5}$ of $\frac{2}{3}$. Let's start by getting everything into a common denominator.
$\frac{2}{3} = \frac{2\times 65}{3\times 65} = \frac{130}{195}$
$\frac{1}{5} = \frac{1\times 39}{5\times 39} = \frac{39}{195}$

Therefore, we are looking for all possible values of A within the range of:
$\frac{130}{195} - \frac{39}{195} = \frac{91}{195}$ to $\frac{130}{195} + \frac{39}{195} = \frac{169}{195}$.
$\frac{1}{13} = \frac{1\times 15}{13\times 15} = \frac{15}{195}$
So this boils down to, what multiples of 15 are within 91 to 169.
$15 \times 6 = 90$
$15 \times 11 = 165$
So, 7 through 11 satisfy the requirement. There are 5 values total. (You could work this problem with decimals, i.e., doing approximating, but the first time I did that I made mistakes. I think this is a safer way to go.)
Therefore, there are a total of 5 out of the 14 possibilities for A that satisfy the requirements. $\frac{5}{14}$ **Ans.**

23. The graph depicts Speed versus Time for a car trip.


Does this scenario describe the graph?
A. "*A car was going up a hill, then along a level street, and then down the hill*."
Well, this is a speed vs. time graph, not up and down. NO!
Does this scenario describe the graph?
B. "*A car was going forward, then stopped, and then drove backwards.*"
Yikes, what map are they looking at? This is speed vs. time, not direction... I don't think so!
Does this scenario describe the graph?
C. "*A car was speeding up, then advanced at*

a constant speed, and then decreased in speed."
That looks more like it.
Does this scenario describe the graph?
D. "*A car was heading northeast, then went east, and then went southeast.*"
I don't see anything on that graph dealing with direction, do you?
C **Ans.**

24. How many terms of the arithmetic sequence 88, 85, 82, .. appear before the number -17 appears? The sequence is decreasing by 3. Always starting with 88, we would subtract 3 once to get the second term, we would subtract 3 twice to get the third term, we would subtract 3 three times to get the fourth term, etc. How many times would we have to subtract 3 from 88 to get to –17?
$88 - (-17) = 105$, which when divided by 3 is 35. That means that we would subtract 3 thirty-five times to get to –17, and –17 is the thirty-sixth term. So there are 35 terms *before* –17. 35 **Ans.**

25. How many distinct four-digit positive integers are such that the product of their digits equals 12.
$12 = 12 \times 1 \times 1 \times 1$
That won't do; the digits must all be between 0 and 9.
$12 = 6 \times 2 \times 1 \times 1$
$12 = 4 \times 3 \times 1 \times 1$
$12 = 2 \times 2 \times 3 \times 1$
Now that we have the 3 different numbers we need to figure out how many of each we can have.
For a 4 digit number with 2 digits the same we have 6211, 6121, 6112, 2116, 2161, 2611, 1126, 1162, 1621, 1612, 1261, 1216 = 12 choices. (Normally there would be 4! or 24 choices but since two of the digits are the same, you divide by 2.)
Same goes for 4311 and 2231.
12 + 12 + 12 = 36 **Ans.**

26. We have the letters M, A, T and H in the following configuration.

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

So how many ways can we spell MATH? Clearly, we can take the M and go up and down or sideways and spell MATH 4 times.

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

Now come the more interesting ways. We can go up one to the A, then left to the T and H.

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

This is symmetric for 4 more. (subtotal 8)
Go up one to the A, then right to the T, then up to the H for 4 more. (subtotal 12).

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

Starting at the M, we can go up to the A and T and then left to the H. (subtotal 16)

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

We can go left of the M to the A and T, then down to the H for 4 more. (subtotal 20).

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

We can go left of the M to the A, then down to the T and down to the H for 4 more. (subtotal 24).

H

H T H

H T A T H

H T A M A T H

H T A T H

H T H

H

Finally, we can go left of the M to the A, then down to the T and then left to the H for 4 more. (total 28).

H

H T H

H T A T H

H T A M A T H

H T A T H

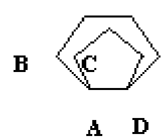
H T H

H

This is all very hard to keep straight, isn't it? Notice, too, that the grid is symmetrical, so if we see that there are 7 ways to spell MATH with the M and the A above it, and we know that there are four "MA" combinations, we can see that there are $4 \times 7 = 28$ ways.

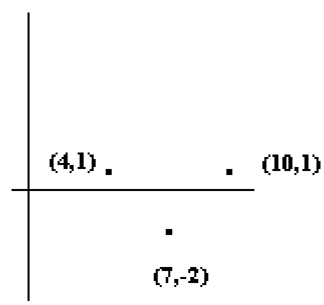
28 **Ans.**

27. A regular pentagon and a regular hexagon are coplanar and share a common side $\overline{AD}$.

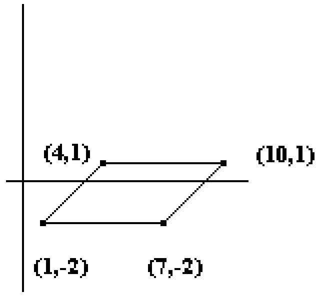


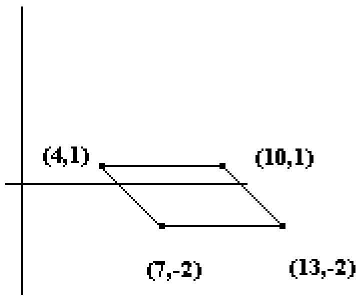
The size of an interior angle of a pentagon, such as angle CAD is 108°. The size of an interior angle of a hexagon, such as angle BAD, is 120°. The degree measure of angle BAC is 120 - 108 = 12. **Ans.**

28. The coordinates of the vertices of a parallelogram are (10, 1), (7, -2), (4, 1) and (x, y).

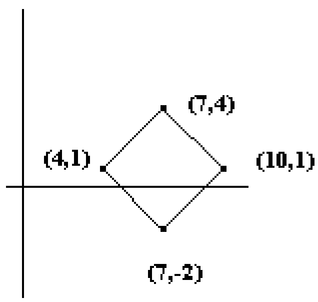


View (10, 1) and (4,1) as being on top. Then (7, -2) and (x, y) will be on the bottom. Either (x,y) will be to the right of (7, -2) or to the left of (7, -2). We already know that y is -2 because the lines have to be parallel. The difference between (10,1) and (4,1) is 6. Therefore, x could be 6 greater than 7 or 6 less than 7.





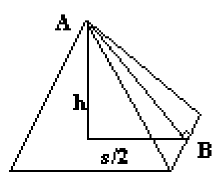
Now, view (7,-2) and (10, -1) as the "bottom" of the parallelogram. Then the x coordinate moves 3 to the right, and the y moves 3 up. 4 + 3 = 7 and 1 + 3 = 4.



1 + 13 + 7 = 21 **Ans.**

29. Camy made a list of every possible distinct five-digit positive even integer that can be formed using each of the digits 1, 3, 4, 5 and 9 exactly once in each integer.
Given that the integer is even and we have only one digit, 4, that is even, all five-digit positive integers must end in 4. Therefore, there are 4! = 24 five-digit positive integers.
If you were to prepare the list of 24 integers for adding, the ones column would only have the digit 4 in it for each integer. The other four columns would have 1, 3, 5 and 9 in them 6 times each.
The ones column would sum to $24 \times 4 = 96$. Therefore a 6 would be in the sum's one's column and we would carry the 9. For the ten's column we have 6 of each of 1, 3, 5 and 9.
1 + 3 + 5 + 9 = 18
$18 \times 6 = 108$
The ten's column would sum to 108 and adding the carry of 9 gives 117. Therefore, there would a 7 in the sum's ten's column and we would carry 11 to the hundred's column.
Similarly, the hundred's column sums to 108 and adding the carry of 11 gives us 119. This gives us a 9 for the sum's hundred's column and another carry of 11 to the thousand's column.
Similarly, the thousand's column sums to 108 and adding the carry of 11 gives us 119. This gives us a 9 for the sum's thousand's column and another carry of 11 to the ten thousand's column.
Similarly, the ten thousand's column sums to 108 and adding the carry of 11 gives us 119. This gives a 9 for the sum's ten thousand's column and another carry of 11. There are no other digits to sum so we end up with:
1,199,976 **Ans.**

30. A right square-based pyramid has a volume of 63,960 cubic meters and a height of 30 meters.
To find the length of $\overline{AB}$ we need to find the side length of the square base of the pyramid.



V = $\frac{1}{3}s^2h$

63,960 = $\frac{1}{3}s^2 \times 30$

63,960 = 10 s^2

6396 = s^2

Let x = $\overline{AB}$

$h^2 + \left(\frac{1}{2}s\right)^2 = x^2$

$h^2 + \frac{1}{4}s^2 = x^2$

900 + 1599 = x^2

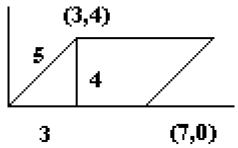
2499 = x^2

2499 is very close to 2500

$\sqrt{2500} = 50$ **Ans.**

TARGET ROUND

1. Each vertex of this parallelogram has integer coordinates. The perimeter is *p* units and the area is *a* square units.



The base and top lengths of the parallelogram are 7. The side of the parallelogram is 5 (3-4-5 right triangle). Therefore:
$p = 2 \times (7 + 5) = 2 \times 12 = 24$
$a = b \times h$
b, or base is 7 and the height is 4.
$a = 7 \times 4 = 28$
$p + a = 24 + 28 = 52$ **Ans.**

2. Marsha adds all but one of the first ten positive integers. The sum of the first ten positive integers is:
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$
$(1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) +$
$(5 + 6) =$
$11 \times 5 = 55$
The sum of all but one of the first ten positive integers is a square, so what number did she remove from the sum? What square is closest to 55? 49 is closest and it is 6 less. Therefore, she removed the 6. **Ans.**

3. What positive five-digit whole number with an 8 in the ten-thousands place is a perfect cube? This question says 80 thousand-something is a perfect cube.
Let's try and get into the ball park.
Take the cube root of 80,000. That value is 43.0886938. This says that 43^3 would be too small. Try 44^3.
$44^3 = 85184$
To be careful, let's also try 45^3.
$45^3 = 91125$
That's too much.
85,184 **Ans.**

4. It takes Darla 8 hours to do a job. It takes Lonnie 6 hours to do the same job.
Therefore, Darla can do $\frac{1}{8}$ of the job in 1 hour and Lonnie can do $\frac{1}{6}$ of the job in 1 hour. Both Darla and Lonnie worked on the job for 3 hours each. This means that they finished:
$3 \times \left(\frac{1}{8}+\frac{1}{6}\right) = 3 \times \left(\frac{3}{24}+\frac{4}{24}\right) =$
$3 \times \frac{7}{24} = \frac{7}{8}$ of the job.
How much of the job is left?
$1 - \frac{7}{8} = \frac{1}{8}$ **Ans.**

5. We have the numbers -2, 5 and -8. We use them to replace the variables, x, y and z in the expression x-yz. How many possible values can we get for the expression?

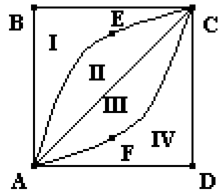
x	y	z		x-yz
-2	5	-8	-2 -(5*-8)	38
-2	-8	5	-2 - (-8*5)	38
5	-2	-8	5 - (-2*-8)	-11
5	-8	-2	5 - (-8*-2)	-11
-8	-2	5	-8 - (-2*5)	2
-8	5	-2	-8 - (5*-2)	2

It's important to note that the product yz is always the same value if you switch y for z and vice-versa. That's why each two lines above result in the same value. Therefore, there are 3 different unique values for x-yz.
$38 + (-11) + 2 = 29$ **Ans.**

6. We are given a chart with the tons of hazardous waste and the costs of disposing that waste for each year. Let's determine the cost per pound for each year by dividing the total weight by the total cost.
In 1995, $\frac{89000}{27.5} = 3236.\overline{36}$.
In 1996, $\frac{54000}{43.5} \approx 1241.37931035$
So far, 1995 is greater.
In 1997, $\frac{197000}{89.5} \approx 2201.117318436$
1995 is still greater.
In 1998, $\frac{168000}{75.5} \approx 2225.165562914$
1995 is still greater.
Finally, in 1999, $\frac{145000}{87.5} \approx 1657.142857143$
1995 is still greater.
I just went through all the computations but you could reduce this quickly by noting that

the division for 1995 was going to start with a 3, while none of the others were.
1995 **Ans**

7. Quadrilateral ABCD is a square.



A circle with center D has arc AEC. Similarly, a circle with center B has arc AFC. AB = 2 cm which means that the radius of both circles is also 2. We need to find the total of areas II and III. Looking at D as the center of a circle, then the areas II + III + IV is actually $\frac{1}{4}$ the area of the circle.

Area II + III + IV = $\frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2)^2 = \pi$

Area I + II + III also is π because the circle with center B is exactly the same size as the circle with center D.

Therefore, all we need do is subtract the area of I + II + III from the area of the entire square to get the area of IV.

The area of the entire square is $2 \times 2 = 4$.

Thus, the area of IV is:

I + II + III + IV - (I + II + III) =

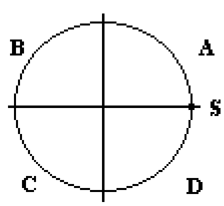
$4 - \pi$

Area I + IV = $2 \times (4 - \pi)$

Area II + III = $4 - (2 \times (4 - \pi))$ =

$4 - 8 + 2\pi = 2\pi - 4 \approx 2.28318 \approx 2.3$ **Ans.**

8. An indoor circular track has a circumference of 50 feet.



Joneal starts at point S, runs in a counterclockwise direction, and then stops when he as run exactly one mile (5280 feet). The track's circumference is 50 feet so we need to divide 5280 by 50 and then see how much is left over.

$\frac{5280}{50}$ = 105R30

Therefore, after 105 revolutions Joneal has 30 feet to go. The circumference is 50 feet and each quarter of the circle is $\frac{50}{4} = 12.5$ feet. So Joneal can run halfway around the circle and use up 25 more feet. That leaves 30 - 25 = 5 feet to go which says that Joneal will stop in the quarter of the circle marked C. **Ans.**

TEAM ROUND

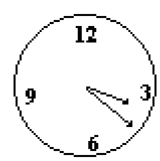
1. The bar graphs shows:

No. States	Year
17	1787-1804
7	1805-1822
2	1823-1840
6	1841-1858
6	1859-1876
6	1877-1894
4	1895-1912
0	1913-1930
0	1931-1948
2	1949-1966

There were 7 + 2 = 9 states chosen between 1805 and 1840. There are a total of 50 states.

$\frac{9}{50}$ **Ans.**

2. The time is 3:20 on a 12-hour analog clock. That looks like this:

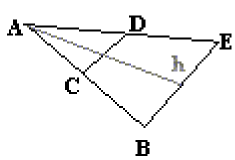


There are 360° in the circle. There are 12 hours. Therefore, each number on the clock (and I didn't show them all) is $\frac{360}{12} = 30°$ from the next. As the minute hand rotates so does the hour hand. 20 minutes after the hour means the hour hand has moved $\frac{20}{60} = \frac{1}{3}$ of the way from the 3 towards the 4 or $\frac{1}{3}$ of 30° or 10°.

Therefore, the hour hand is $(30 \times 3) + 10$ = 100° around from the 12. The minute hand is pointing at the 4 to represent 20 minutes after the hour. This is $40 \times 3 = 120°$ from the 12. Therefore, the difference between the two is 120 - 100 = 20 **Ans.**

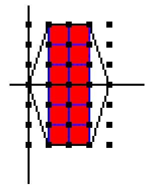
3. Ramon sells two enchiladas and three tacos for \$2.50. He sells three enchiladas and two tacos for \$2.70.
Let e be the cost of the enchilada.
Let t be the cost of the taco.
Write the equation using cents and not dollars so we don't have to deal with decimals.
$2e + 3t = 250$ (Eq. 1)
$3e + 2t = 270$ (Eq. 2)
$6e + 9t = 750$ (Eq. 3 = Eq. 1 $\times$ 3)
$6e + 4t = 540$ (Eq. 4 = Eq. 2 $\times$ 2)
$5t = 210$ (Eq. 4 - Eq. 3)
$t = 42$
We know that 3 enchiladas and 2 tacos cost \$2.70. We now also know that a taco costs \$.42. Therefore, 3 enchiladas and 4 tacos cost:
\$2.70 + (2 $\times$ \$.42) =
\$2.70 + \$.84 = \$3.54 **Ans.**

4. Isosceles triangle ABE has an area of 100 square inches. It is cut by $\overline{CD}$ into an isosceles trapezoid and a smaller isosceles triangle.



The area of the trapezoid is 75 square inches. The altitude of triangle ABE from A is 20 inches. We need to find the number of inches in the length of $\overline{CD}$. First let's find the length of $\overline{BE}$.
$A = \frac{1}{2}bh$
$100 = \frac{1}{2} \times \overline{BE} \times 20$
$100 = 10 \times \overline{BE}$
$\overline{BE} = 10$
The area of triangle ACD is 100 - 75 = 25. Since triangle ACD is similar to triangle ABE and the ratio of their areas is 1:4, we know that the ratio of their corresponding side lengths is 1:2. We know segment BE measures 10, therefore, $\overline{CD} = 5$. **Ans.**

5. What is the number of square units in the area of the hexagon?



You can see that there are 12 square units in red. We are left with two isosceles triangles each of whom has a base of 6 and a height of 1.
$2 \times (\frac{1}{2} \times 1 \times 6) = 6$
$6 + 12 = 18$ **Ans.**

6. Let's number the people from 1 to 15. The first time through every third person will be eliminated so after one round we will have:
1 2 4 5 7 8 10 11 13 14
The next time through numbers 4, 8, 13 will be eliminated but 14 will not yet have gone.
14 1 2 5 7 10 11
Now 2 and 10 are eliminated and 11 has not yet gone leaving us with:
11 14 1 5 7
This time we can eliminate 1 but 5 and 7 have not yet gone.
5 7 11 14
Now 11 leaves and 14 has not yet gone.
14 5 7
7 leaves.
14 5 is left.
14, 5, then 14 so 14 is gone, leaving 5. **Ans.**

7. Katrina and Abbie start a game using a pile of 40 pennies. Each takes anywhere from 1 to 5 pennies from the pile. The player who takes the last penny from the pile wins the game. Abbie plays first. For her to win, she must guarantee that Katrina will be stuck with more than 5 pennies on her last turn. Therefore, if she chooses 4 pennies there will be 36 left. No matter what Katrina does, Abbie can always ensure that there won't be a multiple of 5. 4 **Ans.**

8. Row 1: 2 5
Row 2: 2 7 5
Row 3: 2 9 12 5
The sum of the numbers in row 1 is 7. The sum of the numbers in row 2 is 14. The sum of the numbers in row 3 is 28.
Row 4: 2 11 21 17 5
The sum of the numbers in row 4 is 56.
There's a pattern here. Each sum is twice the sum of the last row.

Row 5: 112
Row 6: 224
Row 7: 448
Row 8: 896
Row 9: 1792
Row 10: 3584 **Ans.**

9. Two women and two girls are standing together on one side of a lake and wish to cross to the opposite side. The rowboat will carry either one woman or the two girls. To find the minimum number of times the rowboat must cross the lake as long as at least one person is in the boat each way consider:
Trip 1: Both girls go across.
Trip 2: One girl comes back. One girl is on the other shore.
Trip 3: One woman goes across.
Trip 4: The other girl comes back. One woman is on the other shore.
Trip 5: Both girls go across.
Trip 6: One girl comes back. One woman and one girl are on the other shore.
Trip 7: The second woman goes across.
Trip 8: The second girl comes back. Both women are on the other shore.
Trip 9: Both girls go across.
9 **Ans.**

10. Three faces of a right rectangular prism have areas of 48, 49 and 50 square units. A right rectangular prism is just a fancy name for a box which has 3 sides x, y and z. The faces whose areas are 48, 49 and 50 are the result of multiplying x by y, y by z and x by z. The volume of the right rectangular prism is xyz.
$(xy) \times (yz) \times (xz) = 48 \times 49 \times 50$
$x^2y^2z^2 = (xyz)^2 = 117600$
$xyz = \sqrt{117600} \approx 342.9285639896$
$xyz \approx 343$ **Ans.**