
MATHCOUNTS®

2005

■ State Competition ■
Team Round
Problems 1–10

School _____

Chapter _____

Team _____, Captain

Members _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round of the competition consists of ten problems which the team has 20 minutes to complete. Team members may work together to solve the problems. Team members may talk during this section of the competition. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. The team captain must record the answers on his/her problem sheet, and all answers must be complete and legible. Only the team captain's problem sheet will be scored.

Total Correct	Scorer's Initials

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1. The perimeter of a rectangle is 56 meters. The ratio of its length to its width is 4:3. What is the length of a diagonal of the rectangle?

1. _____ meters

2. Five states are listed in the table below along with three of their 1998 populations. If the median of all five of the 1998 state populations is 3,360,853.2 less than the mean, and the population of Nebraska is less than the population of Alabama, which is less than that of Illinois, what is the sum of the populations of Illinois and Nebraska?

2. _____

<u>State</u>	<u>1998 Population</u>
Alabama	4,351,999
Delaware	743,603
Illinois	?
Nebraska	?
Texas	19,759,614

3. The sum of the last four digits of Marian's phone number is 30. How many such four-digit sequences are there?

3. _____ sequences

4. The sum of Angie's age and John's age equals Michael's age. Angie's age is a prime number, but John's age and Michael's age are not prime numbers. The sum of John's age and Michael's age is a prime number, and Angie's age minus John's age is a prime number. One more than half of John's age is a prime number. Angie is the only teenager, and she is older than John. How many years old is John?

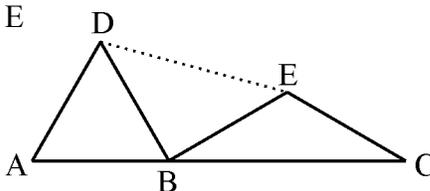
4. _____ years

5. Connie can wash nine windows in one hour. Rick can wash seven windows in one hour. Connie has been washing windows for one hour, when Rick joins her to finish the job. The entire job consists of washing 57 windows. How many hours does Rick work?

5. _____ hours



6. Coplanar points A, B, C, D and E are arranged such that A, B and C are collinear with B between A and C, triangle ABD is equilateral, triangle BEC is isosceles with congruent legs



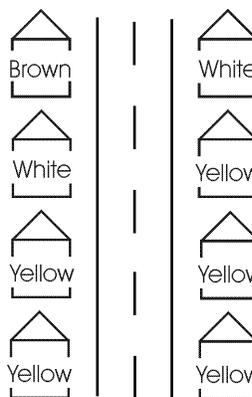
\overline{BE} and \overline{EC} , and points D and E are on the same side of line AC. The measure of $\angle EBC$ is 30 degrees. The areas of triangles ABD and BEC are equal. What is the number of degrees in the measure of $\angle BDE$?

6. _____ degrees

7. When the digits in the number 2005 are reversed we obtain the number 5002, and $5002 = a \cdot b \cdot c$, such that a, b and c are three distinct primes. How many other positive integers are the products of exactly three distinct primes p_1, p_2 and p_3 such that $p_1 + p_2 + p_3 = a + b + c$?

7. _____ integers

8. Each of eight houses on a street is painted brown, yellow or white. Each house is painted only one color and each color is used on at least one house. No two colors are used to paint the same number of houses. In how many ways could the eight houses on the street be painted? One such way is shown here.



8. _____ ways

9. Let line t be the line represented by $3x + 4y = 5$ and let line p be the line perpendicular to line t and containing the point $(5, 5)$. What is the x -coordinate of the point common to line t and line p ? Express your answer as a common fraction.

9. _____

10. In how many different ways can $\frac{2}{15}$ be represented as $\frac{1}{a} + \frac{1}{b}$, if a and b are positive integers with $a \geq b$?

10. _____ ways