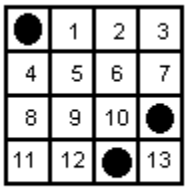
**2006**
**MATHCOUNTS CHAPTER**
**SPRINT ROUND**

1. We have two identical blue boxes and three identical red boxes. The two blue boxes together weigh the same as the three red boxes. The red boxes each weigh 10 ounces. Therefore, the three red boxes weigh 10 × 3 = 30 ounces. The two blue boxes weigh the same, i.e., 30 ounces. Therefore, one blue box weighs $\frac{30}{2}$ = 15. **Ans.**

2. Three black chips have been placed on the game board as shown.




We are asked to find the number of the square where a fourth chip should be placed so that it shares neither a row nor a column with any of the existing chips. Rows 1, 3 and 4 have chips. Therefore, one must be in Row 2. Columns 1, 3, and 4 have chips. Therefore, one must be in Column 2. The number in Row 2, Column 2, is 5. **Ans.**

3. We are asked to find the greatest number of cubes, with edge length 1 inch, which can be placed into a rectangular box measuring 3 inches by 3 inches by 9 inches. Clearly, it is not desirable to have any spaces. We can fill the bottom of the rectangle with 3 rows of 9 cubes each. We can also have a total of 3 of these sets of 3 rows of 9 cubes. So what we are computing here is simply the volume.
V = lwh = 9 × 3 × 3 = 27 × 3 = 81
**Ans.**

4. $6 \times \frac{1}{7} = \frac{6}{7}$ **Ans.**

5. Krista put 1 cent into her new bank on Sunday. On Monday she put 2 cents in and on Tuesday she put 4 cents in. Each day she doubles the amount that she put in the day before. We must find the day of the week that the total amount of money in her bank first exceeded $2. Let's look at the first couple of days…

| **DAY** | **AMOUNT** | **TOTAL** |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 3 |
| 3 | 4 | 7 |
| 4 | 8 | 15 |

Day 1 has a total of $2^1 - 1 = 2 - 1 =$ 1 cent.
Day 2 has a total of $2^2 - 1 = 4 - 1 =$ 3 cents.
Day 3 has a total of $2^3 - 1 = 8 - 1 =$ 7 cents.
Day 4 has a total of $2^4 - 1 = 16 - 1$ = 15 cents.
There is a pattern here. (It's a good thing to recognize powers of 2 and one less than each power of 2!) Now we only need to find the first power of 2 that is greater than 200 to find the number of days. Starting with the power 1, we have 2, 4, 8, 16, 32, 64, 128, 256. So $2^8 - 1 = 256 - 1 = 255$ and it takes 8 days, starting with Sunday, for us to get over 200 cents. Since Sunday is the first day, it must also be the eighth day. Sunday **Ans.**

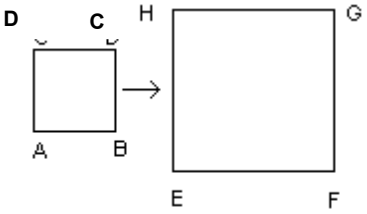
6. We are given a table of positions, how many people have these positions and how much they are paid.

| **TITLE** | **#** | **SALARY** |
|---|---|---|
| Pres. | 1 | $130,000 |
| Vice-Pres. | 5 | $ 90,000 |
| Director | 10 | $ 75,000 |
| Assoc. Dir. | 6 | $ 50,000 |
| Admin. Spec. | 37 | $ 23,000 |

To find the median salary value, first determine the number of employees.
1 + 5 + 10 + 6 + 37 = 59
The median is the middle one or

number 30 (i.e., 29 + 29 + 1 = 59) Since there are 37 administrative specialists, and that is the lowest salary, the median is within that number, and the salary is $23,000. **Ans.**

7. Each side of Square ABCD is doubled in length to form Square EFGH.




The perimeter of square EFGH is 40 cm. This makes each side of the square 10 cm. Since square EFGH had each side doubled in length from square ABCD, a side of square ABCD is $\frac{1}{2}$ of 10 or 5. The area of square ABCD is 5 × 5 = 25. **Ans.**

8. The time zones of New York and Denver are different by 2 hours. A train leaves New York at 2 p.m. (New York time) and arrives in Denver 45 hours later. This equivalent to saying that the train leaves New York at noon, Denver time, and arrives in Denver 45 hours later. If it were to arrive in Denver 48 hours later (instead of 45) it would be exactly two days later or noon in Denver time. But since it takes only 45 hours instead of 48, it must arrive three hours earlier, or 9 a.m. **Ans.**

9. We are given a chart of male and female patients and how many have each type of blood. Since we are asked to find what percent of patients with type AB blood are male, we need only consider the column dealing with type AB blood. That column shows 15 males having type AB blood and 5 females having AB blood for a total of 20 people having AB blood.
$\frac{15}{20} = \frac{3}{4} = 75\%$ **Ans.**

10. A type of cat food recommends that a cat have a daily serving of $\frac{1}{3}$ ounce of dry cat food per pound of body weight. A particular cat is fed $3\frac{2}{3}$ ounces of dry food following the recommendations. To find its weight, just determine how many thirds are in $3\frac{2}{3}$. $3\frac{2}{3} = \frac{11}{3}$
Thus, the cat must weigh 11 pounds. 11 **Ans.**

11. Roger has exactly one of each of the first 22 states' new U.S. quarters. The quarters were released in the same order that the states joined the union and the graph shows the number of states that joined the union in each decade. The graph shows that 12 states joined the union in the decade 1780-1789. Therefore, $\frac{12}{22} = \frac{6}{11}$ of Roger's quarters are from states that joined the union in the decade 1780-1789. $\frac{6}{11}$ **Ans.**

12. We are given the sequence:
0, 1, 1, 3, 6, 9, 27, …
0 is the first term and each subsequent term is produced by alternately adding and multiplying by each successive integer beginning with 1. Thus 1 = 0 + 1.
1 = 1 × 1
3 = 1 + 2
6 = 3 × 2
9 = 6 + 3
27 = 9 × 3
What is the value of the first term that is greater than 125? This shouldn't take long…just continue…
31 = 27 + 4

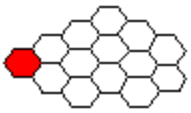
124 = 31 × 4 (gotta go one more!!!)
129 = 124 + 5
129 **Ans.**

13. Carolyn, Julie and Roberta share $77 in a ratio of 4:2:1, respectively. We are asked to determine how much money Carolyn received. Let x = the amount of money Roberta received. Then:
4x + 2x + 1x = 77
7x = 77
x = 11
Carolyn had 4 times as much as Roberta. 4 × 11 = 44 **Ans.**

14. The weight A is balanced by the four weights, 9, 3, 3, and 1. B is balanced by the 5 weights 9, 9, 3, 3, and 1. We are asked to find the minimum number of weights it would take to balance the total weight of A + B where the available weights are 1, 1, 3, 3, 9, 9, 27 and 27. The weight of A is 9 + 3 + 3 + 1 = 16. The weight of B is 9 + 9 + 3 + 3 + 1 = 25. Thus, the weight of A + B = 16 + 25 = 41. To find the smallest number of weights used as the equivalent, start by using the most weights of 27. 41 – 27 = 14
We can use one weight of 27. We can also use 1 weight of 9 leaving 14 – 9 or 5 to go. 5 can be handled by 1 weight of 3 and 2 weights of 1 for a total of 1 + 1 + 1 + 2 = 5 weights. Can we do any better? Replacing a larger weight with smaller weights (even if we had enough) would only add more weights to the total.
5 **Ans.**

15. a = 2, b = 3, c = 4
$(b - c)^2 + a(b + c) =$
$(3 - 4)^2 + 2(3 + 4) =$
$(-1)^2 + 2(7) = 1 + 14 = 15$ **Ans.**

16. The first red hexagon (very dark grey in black & white) touches the next two hexagons as in the following picture.



For the second column, certainly red may not be one of the choices. Choose yellow (light grey in black & white) on top and green (medium grey in black & white) on the bottom. Then all the rest of the colors are pre-determined. Look at the following drawing:

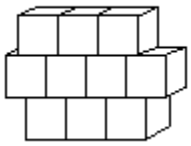
You see that green has colored the top hexagon in the third column. Suppose we'd chosen red. Then green would have to be either the second or third hexagon and the yellow the other. In any event, we'd have green touching green which is illegal. Now suppose we'd chosen yellow for the top hexagon. Then we have choices of red and green for the second and third hexagons and again, we'd violate the rules because we'd still have green touching green. Similarly, for the rest of the hexagons we have a pre-determined set of colors. In the same way, if we go back and choose green on top and yellow on bottom for the second column of hexagons, as in the following picture,

all the rest of the hexagons are pre-determined. 2 **Ans.**

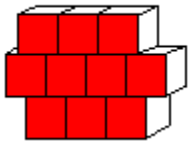
17. One quart of paint is exactly enough for two coats of paint on a 9-foot by 10-foot wall. This means that one quart of paint covers 9 × 10 × 2 = 180 square feet. A 10-foot by 12-foot wall is 10 × 12 = 120 square feet.

$\frac{120}{180} = \frac{12}{18} = \frac{2}{3}$ **Ans.**

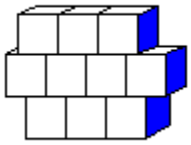
18. Ten unit cubes are glued together as shown.



We are asked to find the surface area. Since these are unit cubes, there is an area of 1 for each cube face. Start by counting up the number of cube faces in the front. These are shown in red in the next figure.



There are 3 + 4 + 3 = 10 cube faces in the front. Similarly, there are 10 cube faces in the back. Now count up the number of cube faces visible from the right as shown in blue in the figure below.



There are 3. Similarly, there are 3 on the left. Now count up how many are shown from the top. These are shown in green in the next figure.

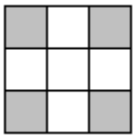
There are 3 on top and then fragments to the left and right of the top row. But these two must add up to 1 cube face since the top row's 3 cube faces are sitting on 4 cube faces of the second row. 3 + 1 = 4.

Similarly, the bottom also shows 4 cube faces.
10 + 10 + 4 + 4 + 3 + 3 = 34 **Ans.**

19. A graph shows the total distance Sam drove from 6 a.m. to 11 a.m. Though the graph shows the following information:
6 a.m. – 0 miles
7 a.m. – 40 miles
8 a.m. – 60 miles
9.a.m. – 100 miles
10 a.m. – 120 miles
11 a.m. – 160 miles, the most important thing to see is that Sam drove 160 miles in 5 hours. His average speed is
$\frac{160}{5}$ = 32 **Ans.**

20. A stock loses 10% of its value on Monday. On Tuesday it loses 20% of the value it had at the end of the day on Monday. Let x = the cost of the stock on Monday morning. At the end of Monday, it has lost 10% of its price or 0.1x.
x – 0.1x = 0.9x
On Tuesday it lost 20% of the price at the end of Monday.
0.2 × 0.9x = 0.18x
0.9x – 0.18x = 0.72x
Thus, the stock now costs 72% of what it did on Monday morning and has lost 100% - 72% = 28% of its value. 28 **Ans.**

21. The numbers 1, 2, and 3 are written in nine unit squares according to the following rules:
-- Each of the numbers appears three times and only one number is placed in each of the nine unit square.
-- Each number is in a unit square horizontally or vertically adjacent to a unit square with the same number.
-- The sum of the numbers in the leftmost column and the sum of the numbers in the top row are each 7.



The second rule states that a number must be next to (horizontally or vertically) the same number and the third rule states that the sum of the numbers in the leftmost column and in the top row are each 7.
7 = 3 + 3 + 1
7 = 3 + 2 + 2
Place the first one in the first column and the second one in the first row and everything falls out.

| | | |
|---|---|---|
| 3 | 2 | 2 |
| 3 | 3 | 2 |
| 1 | 1 | 1 |

3 + 2 + 1 + 1 = 7 **Ans.**

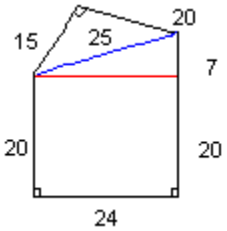
22. Five balls are numbered 1-5 and placed in a bowl. Josh chooses a ball, looks at it and then puts it back in the bowl. Then he chooses a ball again. We are asked to determine the probability that the product of the two numbers will be even and greater than 10. There are 5 × 5 = 25 combinations. Of these, the even products greater than 10 are:
3 × 4 = 12
4 × 3 = 12
4 × 4 = 16
4 × 5 = 20
5 × 4 = 20
or 5 different combinations (actually only 3 combinations but 2 of them can be switched around).
$\frac{5}{25} = \frac{1}{5}$ **Ans.**

23. When a certain negative number is multiplied by six, the result is the same as 20 less than the original number.
Let x be the original number. Then:
6x = x – 20
5x = -20
x = -4 **Ans.**

24. Splitting 2004 into 200 and 4 yields two integers with a common factor greater than 1. The same holds for 2005 and 2006. We are asked to find the first odd-numbered year after 2006 that has this property.
Consider 200 and 7. 7 is not a divisor of 200.
Consider 200 and 9. 9 = 3 × 3 and 3 is not a divisor of 200. If we move on to 2011 we have 201 and 1. Clearly this doesn't satisfy, but looking at 201 says that 3 must be a factor (2 + 0 + 1 = 3 and 3 is divisible by 3) so 2013 will satisfy our requirements. 2013 **Ans.**

25. Two consecutive even numbers are each squared. The difference of the squares is 60.
Let x = the first even number.
Let x+2 = the second even number.
Then $(x + 2)^2 – x^2 = 60$
$x^2 + 4x + 4 – x^2 = 60$
4x = 56
x = 14
14 + 14 + 2 = 30
**Note that they didn't specify which square was subtracted from the other square so we have to try it the other way.
$x^2 – (x + 2)^2 = 60$
$x^2 – (x^2 + 4x + 4) = 60$
$x^2 – x^2 - 4x - 4 = 60$
-4x -4 = 60
-4x = 64
4x = -64
x = -16
-16 + -16 + 2 = -30
Thus, both 30 and -30 are answers.
**30; –30; or 30 and –30 were accepted. **Ans.**

26. We are asked to find the area of the pentagon shown here with sides of length 15, 20, 27, 24, and 20 units.




First draw a horizontal line (in red)

creating the rectangle of dimensions 20 × 24. The area of this rectangle is:
20 × 24 = 480
Next, draw a line from the top of vertical line on the left hand side to the vertex where the lines of length 27 and 20 meet (the blue line). This gives us a right triangle with two sides of 15 and 20. Since this is obviously a multiple of a 3, 4, 5 right triangle, the hypotenuse is 25. The area of the 15, 20, 25 right triangle is:
½ × 15 × 20 = 150
This leaves one more right triangle whose hypotenuse is 25 and one of whose sides is 7. If you don't know about 7, 24, 25 right triangles, just work it out.
$7^2 + x^2 = 25^2$
$49 + x^2 = 625$
$x^2 = 625 - 49 = 576$
$x = 24$
The area of the 7, 24, 25 right triangle is:
½ × 7 × 24 = 84
Totaling everything up:
480 + 150 + 84 =
630 + 84 = 714 **Ans.**

27. I start my bike ride at 20 miles per hour and later continue at only 12 miles per hour. I travel a total of 122 miles in 8 hours. How long did I feel good (i.e., travel at 20 miles per hour)?
Let x = the number of hours I traveled at 20 miles per hour.
Then 8-x is the number of hours I traveled at 12 miles per hour.
$20x + 12(8 - x) = 122$
$20x + 96 - 12x = 122$
$8x = 26$
$x = \frac{26}{8} = \frac{13}{4}$ **Ans.**

28. Charlie made a list of the page number of the last page he finished reading each day. Summing those numbers after 8 days came up to 432 pages. Charlie read the same amount of pages each day.
Let x be the number of pages that Charlie read each day. Then:
$x + 2x + 3x + 4x + 5x + 6x + 7x + 8x = 432$
$36x = 432$
$x = 12$
So Charlie read 12 pages per day.
12 × 8 = 96 **Ans.**

29. We are given that each distinct letter in the equation MATH = COU + NTS is replaced by a different digit chosen from 1 through 9 in such a way that the resulting equation is true.

```
  C O U
+ N T S
-------
M A T H
```

H = 4 and we are asked to find the value of the greater of C and N.
C + N (+ a possible carry) gives us a value of A ones and M tens. M can only be 1 since the maximum value of two digits and a carry is 19.

```
  C O U
+ N T S
-------
1 A T 4
```

Now look at O + T (+ a possible carry) = T If O were 0 (zero) then O + T = T but O must be some digit between 1 and 9. Therefore we must have a carry.
O + 1 + T = T
There will have to be a carry here for this to make sense.
O + 1 + T = T + 10
O = 9

```
  C 9 U
+ N T S
-------
1 A T 4
```

Now let's deal with U and S.
U + S = 14
14 = 9 + 5
14 = 8 + 6
14 = 7 + 7
9 + 5 doesn't work since O is already 9 and each letter corresponds to a unique digit. 7 + 7 doesn't work for the same reason.
So we know that U = 8 and S = 6 or vice versa; it doesn't really matter.

```
  C 9 8
+ N T 6
-------
1 A T 4
```

So, what numbers are left? Just 2, 3, 5, and 7. Clearly C + N + 1 > 10
We have the sum of two numbers plus 1 greater than 10. It could be 3 + 7 + 1 or

5 + 7 + 1. In any case the greatest value of C or N must be 7. **Ans.**

30. A gear turns $33\frac{1}{3}$ times in a minute. Another gear turns 45 times a minute. The two gears start with a mark on each pointing north. We are asked to determine how many seconds will it take before the two gears next have both their marks pointing due north. The first gear has its mark face north every

$$\frac{60}{33\frac{1}{3}} = \frac{60}{\frac{100}{3}} = 60 \times \frac{3}{100} = \frac{180}{100} =$$

$\frac{9}{5}$ seconds.

The second gear has its mark face north every $\frac{60}{45} = \frac{4}{3}$ seconds.

Put these two values into the same denominator.

$$\frac{9}{5} = \frac{27}{15}$$

$$\frac{4}{3} = \frac{20}{15}$$

We need to find the least common multiple of 27 and 20.
Neither number has any factor in common.
27 = 3 × 3 × 3
20 = 2 × 2 × 5
Therefore, the LCM is 27 × 20 = 540

$\frac{540}{15} = 36$ **Ans.**

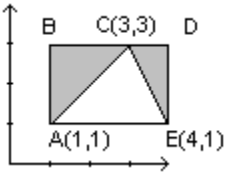
## TARGET ROUND

1. Brass contains 80% copper and 20% zinc. Henri's brass trumpet contains 48 ounces of copper. We need to determine how much zinc. The ratio is 80% to 20% or 4 to 1. Therefore, for every 4 ounces of copper there must be 1 ounce of zinc.

$\frac{48}{4}$ = 12 **Ans.**

2. The rules for traveling the maze require us to only follow a path where the score is between 2 and 14. We have the following possibilities but must check after each operation that we stay within the limit.
7 + -2 + 3 + -4 + 5
7 + -2 – 3 – -4 + 5
7 + -2 – 3 + -4 – 5
7 – -2 + 3 – -4 + 5
7 – -2 + 3 + -4 – 5
7 – -2 – 3 – -4 – 5
Starting with the *first possibility*:
7 + -2 = 5; 5 + 3 = 8; 8 – 4 = 4; 4 + 5 = 9
*Second possibility*:
7 + -2 = 5; 5 – 3 = 2 **STOP**; we're not between 2 and 14.
*Third possibility*:
7 + -2 = 5; 5 – 3 = 2; 2 + -4 = -2 **STOP**; we've gone under.
*Fourth possibility*:
7 – -2 = 9; 9 + 3 = 12; 12 – -4 = 16; **STOP**; we've gone over.
*Fifth possibility:*
7 – -2 = 9; 9 + 3 = 12; 12 + -4 = 8; 8 – 5 = 3
*Sixth possibility*:
7 – -2 = 9; 9 – 3 = 6; 6 – -4 = 10; 10 – 5 = 5
The three possibilities are 9, 3, and 5. We are asked to find the lowest score and that is 3. **Ans.**

3. In the figure below, side AE of rectangle ABDE is parallel to the x-axis and side BD contains the point C. The vertices of the triangle ACE are given in the figure.




We are asked to find the ratio of the area of triangle ACE to the area of rectangle ABDE. The base of triangle ACE is 4 – 1 = 3. The height of triangle ACE is 3 – 1 = 2.

The area of triangle ACE is:
½ × 3 × 2 = 3
The width of rectangle ABDE is the length of side AE which, as before, is 3. The height of rectangle ABDE is the same as the height of triangle ACE, or 2. The area of rectangle ABDE is 3 × 2 = 6.
$\frac{3}{6} = \frac{1}{2}$ **Ans.**
(But then, any time you have as two points of your triangle two adjacent coordinates of your rectangle and the third point is on the side of the rectangle opposite the base of the triangle, the area of the triangle will always be $\frac{1}{2}$ of the area of the rectangle!)

4. The dimensions of A4 paper are 0.21 meters by 0.297 meters. (As an aside, I've actually used this paper!) The area of one sheet of A4 paper is 0.21 × 0.297 = 0.06237 square meters. There are 21 sheets so the total area is 21 × 0.06237 = 1.30977 ≈ 1.3 **Ans.**

5. The arithmetic mean of A, B, and C is 10. This means A + B + C = 30.
A = B – 6
C = B + 3
A + C = 2B – 3
A + B + C = A + C + B = 30 =
(2B – 3) + B
3B – 3 = 30
3B = 33
B = 11
C = B + 3 = 11 + 3 = 14 **Ans.**

6. The values 1 through 26 are assigned to A through Z, respectively. A 9-digit code is created for each letter using prime factorization. The first digit of a letter's code is the number of times 2 is used as a factor; the second digit is the number of times 3 is used as a factor and so on. We are given 6 9-digit codes and asked to determine what word this set of codes spells. The first 9-digit code is:
001000000
The only 1 is in the 3rd column where the 3rd prime, 5, is used as the factor so 5 is the first value or E.
The second 9-digit code is:
000000100
The only 1 is in the 7th column where the 7th prime, 17, is used as the factor so 17 is the second value or Q.
The third 9-digit code is:
010100000
The 1's are the second and fourth prime or 3 and 7.
3 × 7 = 21 or U
The fourth 9-digit code is:
000000000
The only number with no prime factors is 1 or A.
The fifth 9-digit code is:
210000000
$2^2 \times 3 = 4 \times 3 = 12$ or L
The sixth 9-digit code is:
000000010
The only 1 is in the 8th column where the 8th prime, 19, is used as the factor so 19 is value or S
EQUALS **Ans.**

7. A quiz has 15 easy questions and 15 hard questions. Easy questions are worth 4 points each and hard questions are worth 10 points each. Sam earns 92 points on the quiz. We are asked to find the greatest number of hard questions that he could have answered correctly. First ask what is the largest multiple of 10 that is less than 92? Certainly this is 9. But that means he'd get 90 points from the hard ones leaving 2 points for the easy ones. Since an easy question is 4 points this won't work. Try 8 hard ones. Then there are 92 – 80 = 12 points or 3 easy questions. 8 **Ans.**

8. Eli throws five darts at a circular target as shown and each one lands within one of the four regions.

The point values of the red (outer), blue, green and yellow (inner) areas are 1, 2, 4, and 6 points, respectively. We are asked to find the least score greater than five points that is not possible when the point values of the five darts are added together.
What values can we make with only 5 darts?
6 = 1 + 1 + 1 + 1 + 2
7 = 1 + 1 + 1 + 2 + 2
8 = 1 + 1 + 2 + 2 + 2
9 = 1 + 2 + 2 + 2 + 2
10 = 2 + 2 + 2 + 2 + 2
11 = 4 + 1 + 2 + 2 + 2
12 = 4 + 2 + 2 + 2 + 2
13 = 4 + 4 + 1 + 2 + 2
14 = 4 + 4 + 2 + 2 + 2
15 = 4 + 4 + 4 + 1 + 2
16 = 4 + 4 + 4 + 2 + 2
17 = 4 + 4 + 4 + 4 + 1
18 = 4 + 4 + 4 + 4 + 2
19 = 4 + 4 + 4 + 1 + 6
20 = 4 + 4 + 4 + 2 + 6
21 = 4 + 4 + 1 + 6 + 6
22 = 4 + 4 + 2 + 6 + 6
23 = 4 + 1 + 6 + 6 + 6
24 = 4 + 2 + 6 + 6 + 6
25 = 1 + 6 + 6 + 6 + 6
26 = 2 + 6 + 6 + 6 + 6
27 = ?
27 is the first one that we can't make. Note that there is a limit to the checking. We can never get to a point value of 31.

**Note:** A pattern emerges making the counting easier. Every 1 can change to a 2 to create the next number; then every 2,2 can change to a 4,1 for the next number; every 4,1 can change to a 4,2 for the next number, every 4,2 can change to a 6,1 for the next number; and every 6,1 can change to a 6, 2 for the next number. Seeing this pattern makes the listing of the possibilities easier.
27 **Ans.**

## TEAM ROUND

1. In 1992 1200 lire was the same as $1.50. Therefore 1 lire was the same as:

$$\frac{1200}{1.5} = \frac{1}{x}$$

$1200x = 1.5$

$$x = \frac{1.5}{1200} = 0.00125$$

1 lire is the same as $0.00125.
1,000,000 lire is the same as
$0.00125 × 1,000,000 = $1250 **Ans.**

2. A ball bounces back up $\frac{2}{3}$ of the height from which it falls. We are asked to determine after how many bounces the ball first rises less than 30 cm if it is dropped from a height of 243 cm. So consider dropping it from a height of 243 cm. After the first bounce it comes back up to:

$$243 \times \frac{2}{3} = 162 \text{ cm}$$

After the second bounce it comes back up to:

$$162 \times \frac{2}{3} = 108 \text{ cm}$$

After the third bounce it comes back up to:

$$108 \times \frac{2}{3} = 72 \text{ cm}$$

After the fourth bounce it comes back up to:

$$72 \times \frac{2}{3} = 48 \text{ cm}$$

After the fifth bounce it comes back up to:

$$48 \times \frac{2}{3} = 32 \text{ cm}$$

And, after the sixth bounce it comes back up to:

$32 \times \frac{2}{3} = 21\frac{1}{3}$ cm

So it's on the sixth bounce that the ball first rises less than 30 cm. 6 **Ans.**

3. In 2003, the Dodonpa roller coaster had a maximum speed of 106.9 miles per hour. We are asked to look at the graph and determine how many roller coasters had a maximum speed that was faster than the Dodonpa roller coaster. Looking at the graph we see the Dodonpa roller coaster at a height of about 175 feet. There is only one more point to the right at about 120+ miles per hour at around 400 feet. 1 **Ans.**

4. Marika purchased her house with a loan for 80% of the price and paid the remaining $49,400 with her savings. This $49,400 was 100% - 80% = 20% of the purchase price (or $\frac{1}{5}$)

   $49400 \times 5 = 247{,}000$ **Ans.**

5. One interior angle of a convex polygon is 160 degrees. The rest of the interior angles of the polygon are each 112 degrees. The number of degrees in a polygon of n sides or n angles is:

   $180 \times (n - 2)$

   $160 + (112 \times (n - 1)) = 180 \times (n - 2)$

   $160 + 112n - 112 = 180n - 360$

   $180n - 360 = 112n + 48$

   $180n - 112n = 360 + 48$

   $68n = 408$

   $n = \frac{408}{68} = 6$ **Ans.**

6. A 25 passenger bus rents for $110. A 40 passenger bus rents for $170. We are asked to find the minimum cost for renting enough buses for a school trip with 475 passengers.

   A 25 passenger bus costs:

   $\frac{110}{25}$ = $4.40 per person

   a 40 passenger bus costs:

   $\frac{170}{40} = \frac{17}{4}$ = $4.25 per person

   The bigger bus is cheaper per person. Try using as many of those as possible.

   $\frac{475}{40}$ = 11 big buses + 35 people

   We'd need 12 buses but we'd have some empty seats.

   12 × 170 = $2040

   What about using ten 40 passenger buses and three 25 passenger buses so we don't have any empty seats…

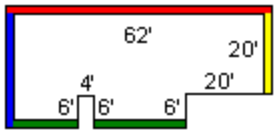
   (10 × 170) + (3 × 110) =

   1700 + 330 = $2030

   There's no way we can do any better because we'll end up using more of the more expensive buses.

   2030 **Ans.**

7. Arpan and Tomika want to place straw in the flower beds surrounding their house. Every angle is a right angle. The shaded regions represent the flower beds.




The width of the flower beds is 2 feet. Each bale of straw costs $2.75 and covers 9 square feet of ground. We are asked to calculate how much it will cost to cover the flower beds with straw if only whole bales of straw may be purchased.

First, we have to figure out how many square feet of flower bed we have. The top part of the flower bed is 62 feet in width but you also have to add on the 2 feet in width on either side. Thus, the area of the flower bed shaded in red above is

$(62 + 2 + 2) \times 2 = 66 \times 2 = 132$

Next is the left vertical part of the bed. How long is that? Well, looking at the right side (by the yellow colored flower bed), you can see that this part is 20 feet. But also by the green part you can see that there is an additional 6 feet of vertical length so the left side is 20 + 6 = 26 feet but you also have to add an additional 2 feet for the flower bed edge. Thus, the area of the blue part is:
(26 + 2) × 2 = 28 × 2 = 56
Next is the right vertical part. This is easier. Only the top is extra flower bed but that was accounted for in the red section. Therefore the area of the yellow section is just:
20 × 2 = 40
Finally how much area is in the green portion? Well, we know it's two feet wide but what about the length? Start with the knowledge that the top side is 62 feet long. The bottom is just like that, **but** there are 20 feet missing on the right and 4 feet missing near the left.
62 – (20 + 4) = 62 – 24 = 38
Thus, while we don't know how much of the 38 feet is in the left part colored green or in the right part, it doesn't matter. This is equivalent to a rectangle of 38 × 2, so the total area is 76 square feet.
The total square footage is:
132 + 56 + 40 + 76 = 304

$\frac{304}{9}$ = 33.77777777777777777

bales of straw. Since we must buy whole bales we will need 34 bales.
The straw costs 34 × $2.75 = $93.50
With 6% tax this is $93.50 × 1.06 = $99.11 **Ans.**

8. A "value Meal" consists of one selection from the entrée menu, drink menu and dessert menu below. The value meal price is calculated by summing up the prices of the three individual items and subtracting 20 cents from the total. The customer orders exactly one of each of the three most expensive value meal combinations and we are asked to find the total cost of the three meals. Here are the menus:

**Entrée:**
Hot dog: $1.25
Hamburger: $1.65
Chicken: $1.80
Pizza: $2.25

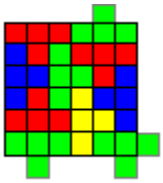
**Drink:**
Coffee: $0.75
Soda: $1.05
Juice: $1.25

**Dessert:**
Cookie: $0.65
Pudding: $1.30

To find the most expensive meal, let's take the most expensive item from each category.
That would be pizza, juice and pudding for a total of:
2.25 + 1.25 + 1.30 – 0.20 (for the "value" ) = 4.60
What's the next most expensive meal? Well, if we go from pizza to chicken it would be 45 cents cheaper. If we go from juice to soda, it would be 20 cents cheaper. If we go from pudding to cookie, it would be 65 cents cheaper. Go with the soda, so the second most expensive meal is pizza, soda and pudding.
2.25 + 1.05 + 1.30 – 0.20 = 4.40

Finally the third most expensive meal.
If we go with chicken, it would be 45 cents cheaper. If we go with coffee, it would be 30 cents cheaper and if we go with cookie it would be 65 cents cheaper. So maybe the coffee? Well, that would make this meal a total of 20 + 30 = 50 cents cheaper than the most expensive meal. But if we switch in chicken for pizza and go back up to the juice the meal would only be 45 cents cheaper so the third most expensive meal is chicken, juice and pudding.
1.80 + 1.25 + 1.30 – 0.20 = 4.15
Thus, the total value for all three meals is:
4.60 + 4.40 + 4.15 = 13.15 **Ans.**

9. A 6 by 6 grid of 36 unit squares is completely covered in T-shaped pieces consisting of four unit squares. The pieces can not overlap but may extend over the side of the 6 by 6 grid. We are asked to find the fewest number of pieces required to cover the grid. Since these pieces are T-shaped the best we can do is to try and interweave them as much as possible and leave the number of portions extending outside the grid as the absolute minimum. The following grid illustrates how to do that.



The way to see this (without trying to do the drawing first) is as follows: The 6 by 6 grid has 36 unit squares in it. Each of the T-shaped pieces occupies 4 squares.

$\frac{36}{4}$ = 9 pieces but we know we're going to extend over the edge so the best we can shoot for is 10 and we've proven we can do that. 10 **Ans.**

10. Three friends have a full bag of jelly beans. Mike took $\frac{1}{3}$ of the jelly beans in the full bag. Zac took $\frac{1}{2}$ of the jelly beans in the full bag. Kary took what was left. Mike ate $\frac{1}{2}$ of his jelly beans, Zac ate $\frac{1}{3}$ of his jellybeans and Kary ate all of hers. Mike and Zac now have a total of 45 jellybeans together and we re asked to find out how many jellybeans Kary ate. Mike had $\frac{1}{3}$ of the jelly beans and then ate $\frac{1}{2}$ of them so he actually had $\frac{1}{6}$ of the entire original bag of jelly beans left. Zac had $\frac{1}{2}$ of the bag and ate $\frac{1}{3}$ of them leaving $\frac{1}{2} - \left(\frac{1}{3} \times \frac{1}{2}\right) = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ of the original bag of jelly beans left. Between the two of them they now have 45 jellybeans which must be $\frac{1}{2}$ of the bag of jellybeans. Thus, the total number of jellybeans that were originally in the bag was: 45 × 2 = 90 jellybeans. Between Mike and Zac they originally had $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ of the bag leaving just $\frac{1}{6}$ of the bag for Kary.

$\frac{1}{6}$ × 90 = 15 **Ans.**