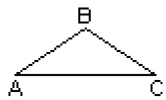
2007
MATHCOUNTS CHAPTER COMPETITION

SPRINT ROUND

1. Triangle ABC is an obtuse, isosceles triangle. Angle A measures 20 degrees. We are asked to find the measure of the largest interior angle of the triangle, which is angle B.



Isosceles triangles have 2 angles that are the same, in this case angle A and angle C.
B = 180 – (A + C)
B = 180 – (20 + 20)
B = 180 – 40 = 140 **Ans.**

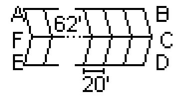
2. Quentin spent $480 to purchase 30 books. We are asked to determine how much 45 books cost if we use the average price per book. We don't need to determine what the average price per book is, though. If 30 books are $480 then 15 books are just half of that or $240.
30 books + 15 books = 45 books
480 + 240 = 720 **Ans.**

3. 25% of the households earn less that $30,000 per year. 65% of households earn less than $80,000 per year. To find the largest possible percent of households that could earn between $30,000 and $80,000 per year, we already know that 25% earn less than $30,000. Therefore, we must subtract that from the 65%.
65% - 25% = 40% **Ans.**

4. 3000 + x – 2000 = 1500 + 1000
1000 + x = 2500
x = 2500 – 1000 = 1500 **Ans.**

5. How many cubic feet are in one cubic yard? Given that one yard is equal to three feet, we have a cube with sides of 3 feet.
3 × 3 × 3 = 27 **Ans.**

6. 48 congruent parallelograms with sides of length 62 feet and 20 feet are placed in a chevron pattern forming hexagon ABCDEF, as shown. We are asked to find the perimeter of the hexagon ABCDEF.



If there are 48 congruent parallelograms and there are 2 rows of them, then there are 24 parallelograms between A and B. Since the smaller side of the parallelogram is 20 feet,
AB = 24 × 20 = 480
ED = AB = 480
Since the longer side of the parallelogram is 62,
AF = FE = BC = CD = 62
4 × 62 = 248
248 + 480 + 480 = 1208 **Ans.**

7. The mean of four distinct positive integers is 5. If the largest integer is 13, what is the smallest integer? Let x, y, and z be the three integers we don't know.

$$\frac{x+y+z+13}{4} = 5$$

x + y + z + 13 = 20
x + y + z = 20 – 13 = 7
x, y and z must be distinct integers.
7 = 4 + 2 + 1
(anything else will violate the distinctness requirement)
Therefore, the smallest integer is 1. **Ans.**

8. Congruent segments are used to form equilateral triangles in a sequence so that each figure contains one more triangle than the preceding figure.

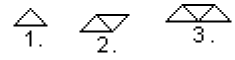


Figure 3 has seven congruent segments and we must determine how many congruent segments are needed to make Figure 25 of the sequence.
Figure 1 uses 3 segments.
Figure 2 uses 3 + 2 = 5 segments.
Figure 3 = 3 + 2 + 2 + 7 segments.

Figure N uses 3 + (2 × (N – 1)) segments.
Figure 25 must use
3 + (2 × (25 -1)) =
3 + (2 × 24) =
3 + 48 = 51 segments.
51 **Ans.**

9. What is the sum of the odd integers from 11 through 39, inclusive?
11 + 13 + … + 37 + 39 =
(11 + 39) + (13 + 37) + … =
$50 \times \frac{15}{2} = 25 \times 15 = 375$ **Ans.**

10. The average amount of money spent by a person at a sporting event in 2000 was $8.00. 75% of this (or $6.00) was for the ticket price. In 2005, the amount spent increased by 50% (or $4.00 for a total of $12.00). However, the ticket price of $6.00 did not change. Therefore, the non-ticket costs are ($12.00 - $6.00) – ($8.00 - $6.00) =
$6.00 - $2.00 = $4.00 **Ans.**

11. The I-Pick-Up service charges $1 per ounce of the packages' weight plus $5 for each distinct drop-off site. After this a 4% service fee is added to the sub-total. Chen Li's order is 1 four-ounce package and 1 two-pound package to Imatrin and 1 eight-pound package to Storyville.
The two packages to Imatrin weigh 4 + (2 × 16) = 4 + 32 = 36 ounces. At $1 per ounce this will cost $36 plus the $5 for the drop-off site.
36 + 5 = 41
The final drop-off is an 8-pound package which is 8 × 16 = 128 ounces. This will cost
128 + 5 = 133
The total cost before the service fee is: 41 + 133 = 174
The 4% service fee is:
174 × 0.04 = 6.96
174 + 6.96 = 180.96 **Ans.**

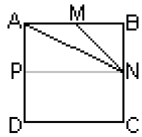
12. A line contains the points (-1,6), (6,k) and (20,3). We are asked to find the value of k.
Using y=mx+b with the two points that do not have k, we get:
6 = (m × -1) + b
6 = -m + b
and
3 = 20m + b
Subtracting,
$3 = -21m$ and $m = -\frac{3}{21} = -\frac{1}{7}$
Substituting back into:
6 = m + b
we get
$6 = \frac{1}{7} + b$
$b = 6 - \frac{1}{7} = \frac{41}{7}$
The equation y = mx + b becomes:
$y = -\frac{1}{7}x + \frac{41}{7}$
or:
7y = -x + 41
Substituting in the point (6,k):
7k = -6 + 41
7k = 35
k = 5 **Ans.**

13. A seven-sided convex polygon has 1 right angle. We are asked to find out how many diagonals this polygon has. The number of diagonals in a convex polygon is:
$$\frac{n \times (n - 3)}{2}$$
For a 7-sided convex polygon the number of diagonals is:
$$\frac{7 \times (7 - 3)}{2} = \frac{7 \times 4}{2} = 14$$ **Ans.**

14. The product of three consecutive odd integers is 1287.
Let's factor 1287 to see if we can come up with 3 consecutive odd integers.
1287 = 3 × 429 = 3 × 3 × 143 =
3 × 3 × 11 × 13 = 9 × 11 × 13
We have to find the sum of these integers.
9 + 11 + 13 = 33 **Ans.**

15. Square ABCD has a point M which is the midpoint of side AB and Point N, which is the

midpoint of side BC. We are asked to find the ratio of the area of triangle AMN to the area of square ABCD.



Let x be the length of a side of square ABCD. Then,

$AM = MB = BN = NC = \frac{x}{2}$.

If we draw a line from N to P, the midpoint of AD, we form rectangle ABNP. Its area is $\frac{1}{2}$ the area of square ABCD or $\frac{x^2}{2}$.

The area of triangle MBN is

$\frac{1}{2} \times MB \times BN =$

$\frac{1}{2} \times \frac{x}{2} \times \frac{x}{2} = \frac{x^2}{8}$

The area of triangle ANP is $\frac{1}{2}$ the area of rectangle ABNP or $\frac{x^2}{4}$.

The area of triangle AMN is the area of rectangle ABNP minus the area of triangle MBN minus the area of triangle ANP.

$\frac{x^2}{2} - \frac{x^2}{8} - \frac{x^2}{4} = \frac{x^2}{8}$

Thus, the ratio of triangle AMN to square ABCD is:

$\frac{\frac{x^2}{8}}{x^2} = \frac{1}{8}$ **Ans.**

16. How many non-congruent triangles are there with sides of integer length having at least one side of length five units and having no side longer than five units? The thing we must remember is that the sum of the two sides of a triangle must always be greater than the third side. So the first triangle is clearly 5,5,5. What about 5, 5, 4?
5 + 5 = 10; 10 > 4
5 + 4 = 9; 9 > 5
5 + 4 = 9; 9 > 5 -- #2
Now 5, 5, 3.
5 + 5 = 10; 10 > 3
5 + 3 = 8; 8 > 5
5 + 3 = 8; 8 > 5 -- #3
Similarly, 5, 5, 2 and 5, 5, 1 will also work. We are up to 5 triangles.
Now use 4 as the second side.
5, 4, 5 is the same as 5, 5, 4.
5, 4, 4?
5 + 4 = 9; 9 > 4
5 + 4 = 9; 9 > 4
4 + 4 = 8; 8 > 5
Similarly, for 5, 4, 3, and 5, 4, 2.
But with 5, 4, 1, we have 4 + 1 = 5 and that is not greater than 5.
So we have 3 more.
5 + 3 = 8
No use 3 as the second side.
5 + 3 + 5 is already done as is 5 + 3 + 4.
5 + 3 + 3 is okay but 5 + 3 + 2 is not nor is 5 + 3 + 1.
We have just one more for a total of 9.
Using 2 as the second value, we have 5, 2, 2 and 5, 2, 1 and neither of those will work nor will 5, 1, 1.
9 **ANS.**

17. $\left(\frac{1}{3} - \frac{1}{9}\right) + \left(\frac{1}{27} - \frac{1}{81}\right) + \frac{1}{243} =$

$\frac{2}{9} + \frac{2}{81} + \frac{1}{243} =$

$\frac{54}{243} + \frac{6}{243} + \frac{1}{243} = \frac{61}{243}$ **Ans.**

18. Chocolate can be packaged in 1, 2 or 4 piece containers, but they must be full. We are asked to find how many different combinations of boxes can be used for the customer's 15 chocolate pieces. Let's start by using boxes with 4 chocolate pieces. Then, we have 2 possibilities: 3 boxes of 4, 1 box of 1 and 1 box of 1(3-1-1) or 3 boxes of 4 and 3 boxes of 1 (3-0-3). Using 2 boxes of 4, we have 4 possibilities: 2-3-1, 2-2-3, 2-1-5, and 2-0-7. Using 1 box of 4, we

have 6 possibilities: 1-5-1, 1-4-3, 1-3-5, 1-2-7, 1-1-9, and 1-0-11. Finally using 0 boxes of 4, we have 8 possibilities: 0-7-1, 0-6-3, 0-5-5, 0-4-7, 0-3-9, 0-2-11, 0-1-13 and 0-0-15. You can see that there is a pattern here, i.e., 2, 4, 6, 8.
2 + 4 + 6 + 8 = 20 **Ans.**

19. The value of **[**x**]** is the greatest integer less than or equal to x. We are asked to find the arithmetic mean of the 10 members in the set.

$[-\pi] = -4 \quad [-\frac{1}{2}] = -1$

$[0] = 0 \quad [\frac{1}{2}] = 0$

$[0.689] = 0 \quad [\frac{\pi}{4}] = 0$

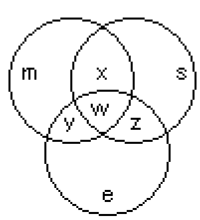
$[\frac{\pi}{3}] = 1 \quad [2] = 2$

$[\sqrt{5}] = 2 \quad [\pi] = 3$

-4 + -1 + 1 + 2 + 2 + 3 =
-5 + 3 + 5 = 3

The mean is: $\frac{3}{10}$ **Ans.**

20. A survey of 100 students doing homework showed that:
59 students did math
49 students did English
42 students did science
20 students did English and science
29 students did science and math
31 students did math and English
12 students did math, science and English
We are asked to find how many students did no math, no English and no science homework. (Just what **were** they doing????)
This calls for a Venn diagram!



Let m = the number of students who do just math homework.
Let s = the number of students who do just science homework.
Let e = the number of students who do just English homework.
Let x be the number of students who do both math and science homework.
Let y be the number of students who do both math and English homework.
Let z be the number of students who do both science and English homework.
Finally, let w be the number of students who do all three types.
w = 12
The number of students doing both math and science are:
x + w = 29
x + 12 = 29
x = 17
The number of students doing both math and English are:
y + w = 31
y + 12 = 31
y = 19
The number of students doing both English and science are:
z + w = 20
z + 12 = 20
z = 8
The number of students doing math homework is:
m + x + y + w = 59
m + 17 + 19 + 12 = 59
m + 48 = 59
m = 11
The number of students doing science is:
s + x + z + w = 42
s + 17 + 8 + 12 = 42
s + 37 = 42
s = 5
Finally, the number of students doing English homework is:
e + y + z + w = 49
e + 19 + 8 + 12 = 49
e + 39 = 49
e = 10
The total number of students doing one or more of English, math and science homework is:
m + s + e + x + y + z + w =
11 + 5 + 10 + 17 + 19 + 8 + 12 = 82
100 – 82 = 18 **Ans.**

21. 6x + y = 15
6x = -y + 15
3x = $-\frac{1}{2}y + \frac{15}{2}$
If 3x = ay + b
then a = $-\frac{1}{2}$
and b = $\frac{15}{2}$
a + b = $-\frac{1}{2} + \frac{15}{2} = \frac{14}{2} = 7$
7 **Ans.**

22. We are given (1,2,3,4,5,6,7,8,9,8,7,6,5,4,3,2,1,2,3, 4,…) and asked to find the 1000th integer in the list. 1 is the first integer in the list is also the 17th integer in the list and will appear every 16 entries in the list after that, i.e., 33 is the next 1. So, the series repeats every 16 entries starting with the first integer in the list.
$\frac{1000}{16}$ = 62 R 8
We are looking for the 8th entry in the list: 8 **Ans.**

23. The three-digit integer N yields a perfect square when divided by 5. When divided by 4, the result is a perfect cube. What is the value of N?
N = $5x^2$
N = $4y^3$
$5x^2 = 4y^3$
$x^2 = \frac{4}{5}y^3$
What square is 80% of what cube?
List the cubes between 1 and 1000:
1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
Remember that y^3 should be a multiple of 5. Of the cubes we listed, 125 and 1000 are multiples of 5. Could 1000 be it? No, since 800 is not a perfect square. But 80% of 125 is a perfect square, i.e., 100.
Therefore, y = 5 and
$4y^3 = 4 \times 5 \times 5 \times 5 = 500$ **Ans.**

24. Grady rides his bike 60% faster than his little brother, Noah. If Grady rides 12 miles further than Noah in two hours, how fast does Noah ride? If Noah covers x miles in 1 hour, then Grady covers 1.6x. Therefore,
(1.6 × 2)x – 2x = 12
3.2x – 2x = 12
1.2x = 12
x = 10 **Ans.**

25. The length of a diagonal of a square is $\sqrt{2} + \sqrt{3}$ square units. We are asked to find the area of the square.
Let x = a side of the square.
Let d = a diagonal of the square.
Then,
$x^2 + x^2 = d^2$
$2x^2 = d^2$
The area of the square is just x^2 so
$x^2 = \frac{d^2}{2} = \frac{\left(\sqrt{2} + \sqrt{3}\right)^2}{2} =$
$\frac{2 + 2\sqrt{6} + 3}{2} = \frac{5 + 2\sqrt{6}}{2} =$
$\frac{5}{2} + \sqrt{6}$ **Ans.**

26. Either increasing the radius or the height of a cylinder by six inches will result in the same volume. Also, the original height of the cylinder is two inches. We are asked to find the original radius. The volume of a cylinder is:
$V = \pi r^2 h$
Let r_1 = the original radius.
Let h_1 = the original height.
Let r_2 = the original radius increased by 6
Let h_2 = the original height increased by 6
We know that
$\pi r_1^2 h_2 = \pi r_2^2 h_1$
$r_1^2 h_2 = r_2^2 h_1$
And we also know that
$h_1 = 2$ so
$h_2 = h_1 + 6 = 2 + 6 = 8$
Substituting back in we have:
$8r_1^2 = 2r_2^2$
$4r_1^2 = r_2^2$

And, finally,
$r_2 = 2r_1$
But just what is r_2? It's just r_1 increased by 6, i.e.,
$r_2 = r_1 + 6$
$r_1 + 6 = 2r_1$
$r_1 = 6$ **Ans.**

27. Consider this pattern

$$\begin{array}{cccc} & & 1 & \\ & 3 & 5 & \\ 7 & 9 & 11 & \\ 13 & 15 & 17 & 19 \\ & \vdots & & \end{array}$$

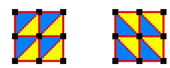
The positive, odd integers are arranged in a triangular formation. Each row has one more entry than the previous row. So what is the sum of the integers in the 15th row?
Uh…why don't we look for a pattern?
The sum of the 1st row is: 1
The sum of the 2nd row is: 8
The sum of the 3rd row is: 27
The sum of the 4th row is: 64
Oh my….
Doesn't this just look like:
$1^3, 2^3, 3^3, 4^3$?
(Never panic until you look for the pattern!!!)
Therefore, the sum of the 15th row is just $15^3 = 3375$ **Ans.**

28. Four couples are at a party. Four people of the eight are randomly selected to win a prize. Given that no person can win more than one prize, what is the probability that both members of at least one couple win a prize?
The simplest way to figure this out is to determine how many ways no couples could win. Note that once you pick the first person, the second person would have to be the other person in that couple. That would leave 6 people. You pick one from the 6 and again, the next person is already known. Similarly you would then have to pick one from 4 and one from 2.

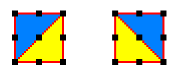
$$\frac{8 \times 6 \times 4 \times 2}{8 \times 7 \times 6 \times 5} = \frac{8}{35}$$

$$1 - \frac{8}{35} = \frac{27}{35}$$ **Ans.**

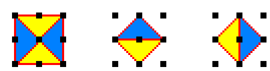
29. How many different sets of three points in this 3 by 3 grid of equally spaced points can be connected to form an isosceles triangle? Oh, I just love these – so many things to draw!! An isosceles triangle means that two sides must be the same.
Let's define as 1 unit the distance between two adjacent points. Then the first set of isosceles triangles are those where the two sides have length 1 unit.



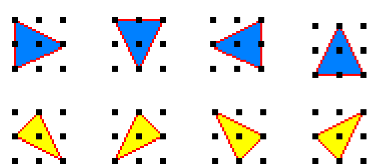
There are 8 × 2 = 16 of them. Next is to look at the set of isosceles triangles where the two sides have a length of 2 units.



There are four of those.
Next we look at all the triangles that have the two sides with lengths of the hypotenuse of the triangles shown above (i.e., $\sqrt{2}$).



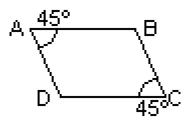
There are eight of those.
Finally, we look at all the triangles that have the two sides with length of the hypotenuse of a right triangle whose other sides are 2 and 1 (i.e., $\sqrt{5}$)

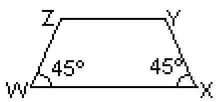


And there are eight of those.
16 + 4 + 8 + 8 = 36 Wasn't that fun?
36 **Ans.**

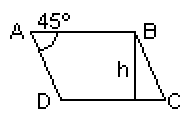
30. In parallelogram ABCD, AB= 16 cm, DA = $3\sqrt{2}$ cm and sides AB and DA form a 45-degree interior angle. In isosceles trapezoid WXYZ with WX ≠ YZ, This means that angle C is

also a 45-degree angle. WX is the longer parallel side and has length 16 cm, and both interior angles measure of 45 degrees. Trapezoid WXYZ has the same area as parallelogram ABCD. We are asked to find the length of YZ. First look at the initial pictures of the parallelogram and trapezoid.





The area of a parallelogram is
A = bh
We certainly have the base of the parallelogram, that's 16 since AB = BC but we need the height. Drop a perpendicular from D=B to DC.



Because that angle is 45º that makes the angle between the height and BC also 45º and we have an isosceles triangle whose hypotenuse just happens to be BC which is $3\sqrt{2}$.

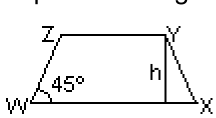
$$h^2 + h^2 = \left(3\sqrt{2}\right)^2$$

$$2h^2 = \left(3\sqrt{2}\right)^2 = 18$$

$h^2 = 9$
h = 3
Thus, the area of the parallelogram is
16 × 3 = 48
Now, remember that we were told that trapezoid WXYZ has the same area as the parallelogram ABCD. The area of the trapezoid is

$$A = \frac{1}{2}\left(b_1 + b_2\right)h$$

We have A, and b_1. We are asked to find b_2 and we don't have the trapezoid's height.



So if we draw another perpendicular, this time from Y to WX, we know that we have gotten another isosceles triangle and YZ + 2h = WX
Since WX = 16
YZ + 2h = 16
YZ = 16 – 2h

$$48 = \frac{1}{2}\left(16 + \left(16 - 2h\right)\right)h$$

$$48 = \frac{1}{2}\left(32 - 2h\right)h$$

$$96 = \left(32 - 2h\right)h$$

$96 = 32h - 2h^2$
$h^2 - 16h + 48 = 0$
(h – 12) × (h – 4) = 0
h = 4
h = 12
If h = 12, then YZ = 16 – 2h = -8.
No. Therefore, h = 4
YZ = 16 – 2h = 16 – 8 = 8 **Ans.**

TARGET ROUND

1. The symbols ♣, ♥, ♦, and ♪ each represent a distinct digit that has not been used already in the subtraction problem below. Whenever a symbol appears more than once, it represents the same digit each time. We are asked to find the digit that ♣ represents.

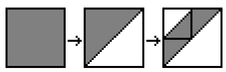
```
   6  ♥  ♦
 - ♣  8  ♦
 ---------
   1  ♪  ♥
```

These always look harder than they really are. First of all, we have in the ones column a diamond subtracted from a diamond which, of course, no matter what the value of the diamond, will leave 0. Therefore, ♥ = 0. Let's rewrite the subtraction problem.

```
   6  0  ♦
 - ♣  8  ♦
 ---------
   1  ♪  0
```

We also know that we don't need to borrow from the tens digit to do the subtraction in the ones column. But we will have to borrow 1 from the hundreds column to do (0 – 8). Therefore, instead of
6 - ♣ = 1, we will have
5 - ♣ = 1 and therefore,
♣ = 4 **Ans.**

2. An 8-inch by 8-inch square is folded along a diagonal creating a triangular region. This resulting triangular region is then folded so that the right angle vertex just meets the midpoint of the hypotenuse. We are asked to find the area of the resulting trapezoidal figure.



After the first fold, the triangular region is $\frac{1}{2}$ the area of the square.
The second fold shows that we essentially created a smaller square whose sides are 4 and removed $\frac{1}{2}$ of it. That's $\frac{1}{4}$ of the area.

$$\frac{1}{2} - \frac{1}{4}\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$$

Thus, the trapezoid's area is $\frac{3}{8}$ of the area of the square. The area of the square is 8 × 8 = 64

$\frac{3}{8} \times 64 = 24$ **Ans.**

3. The distance between two cities on a map is 4 cm. We are asked to find out how far apart the cities are if the scale is 0.5 cm = 1 km.
If 0.5 cm = 1 km, then 1 cm = 2 km.
4 cm = 4 × 2 = 8 km. **Ans.**

4. Six years ago a vacant lot was turned into a park and 46 trees were planted. Three years ago 50 trees were planted and today, 60 trees were planted. If we assume that all of these planted trees were planted as seeds (so their actual age is from date of planting) and that they all survive with no other trees added in the next 10 years, then what will be the average age of the trees in the park 10 years from today?
Let's start with the original 46 trees. 10 years from now, they will have been planted 10 + 6 = 16 years ago.
46 × 16 = 736
The 50 trees that were planted 3 years ago will each be 13 years old 10 years from now.
13 × 50 = 650
The 60 trees that were planted today will each be 10 in 10 years time.
60 × 10 = 600
There are a total of 46 + 50 + 60 trees = 156 trees.
736 + 650 + 600 = 1986

$\frac{1986}{156} \approx 12.73076923 \approx 13$

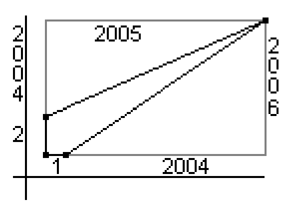
13 **Ans.**

5. The diameter of a circle is 20cm. But that value may be up to 20% off. What is the largest possible percent error in the computed area of the circle?
Suppose the circle is 20% larger.
20 × 0.2 = 4
The diameter of the circle would be 24 and the radius would be 12

$A = \pi r^2 = 144\pi$

If the diameter of the circle was 20% smaller the diameter would be:
20 – 4 = 16
Thus, the radius would be 8 and the area of that circle is 64π.
A circle with diameter of 20 has a radius of 10 and an area of 100π.
If the diameter were 20% larger, then the error in the computed area of the circle is 44%. If the diameter were 20% smaller, the error is 36%.
44% **Ans.**

6. A quadrilateral in the plane has vertices at (1,3), (1,1), (2,1) and (2006,2007). We are asked to find the area of the quadrilateral. [This is a little hard to draw, of course, so the figure is **NOT** drawn to scale!!!]



To find the area of the quadrilateral, draw a rectangle including the points of the quadrilateral. This rectangle has the vertices (1,1), (1,2006), (1,2007) and (2006,2007). This makes the sides of the rectangle 2005 and 2006 respectively and the area is 2005 × 2006 = 4022030

Note that in drawing the rectangle we created two right triangles, the first with two sides of 2004 and 2006 and the second with two sides of 2004 and 2005. If we subtract the areas of those two triangles from our rectangle we'll have the area of the quadrilateral.

$\frac{1}{2}(2004 \times 2006) = \frac{1}{2}(4020024) =$ 2010012

The area of the second triangle is

$\frac{1}{2}(2004 \times 2005) = \frac{1}{2}(4018020) =$ 2009010

2010012 + 2009010 = 4019022

4022030 – 4019022 = 3008 **Ans.**

7. 6 boys and 6 girls are seated randomly in a row of 12 chairs. We are asked to find the probability that no two boys are seated next to one another and no two girls are seated next to one another.

Suppose you choose a boy first. Then you must choose a girl next, then a boy, then a girl etc. Therefore you have a total of:

6 × 6 × 5 × 5 × 4 × 4 × 3 × 3 × 2 × 2 × 1 × 1 = 6! × 6! different combinations. There are 12! possible combinations.

$$\frac{6!6!}{12!} = \frac{6!}{12 \times 11 \times 10 \times 9 \times 8 \times 7} =$$

$$\frac{5 \times 4 \times 3}{11 \times 10 \times 9 \times 8 \times 7} =$$

$$\frac{4 \times 3}{11 \times 2 \times 9 \times 8 \times 7} =$$

$$\frac{3}{11 \times 2 \times 9 \times 2 \times 7} =$$

$$\frac{1}{11 \times 2 \times 3 \times 2 \times 7} = \frac{1}{924}$$

The same is true if you choose a girl first.

$$\frac{1}{924} + \frac{1}{924} = \frac{2}{924} = \frac{1}{462}$$ **Ans.**

8. Dr. Lease leaves his house at exactly 7:20 a.m. every morning. When he averages 45 miles per hour, he arrives at his workplace 5 minutes late. When he averages 63 miles per hour, he arrives five minutes early. We must find the speed that Dr. Lease should average to arrive at his workplace precisely on time.

Let x be the number of minutes that Dr. Lease needs to travel at the right speed to get there precisely on time. When he travels at 45 miles per hour (or $\frac{45}{60} = \frac{3}{4}$ mi. per minute) he travels $\frac{3}{4}(x+5)$ miles. When he travels at 63 miles per hour (or $\frac{63}{60} = \frac{21}{20}$ mi. per minute) he travels $\frac{21}{20}(x-5)$ miles. Of course, these two values are the same, i.e.,

$$\frac{3}{4}(x+5) = \frac{21}{20}(x-5)$$

15(x + 5) = 21(x – 5)

15x + 75 = 21x- 105

6x = 180

x = 30

30 is the number of minutes he must travel and the mileage he must travel is

$$\frac{3}{4}(30+5) = \frac{3}{4} \times 35 = \frac{105}{4} = 26\frac{1}{4}$$

miles

If we divide the distance by the number of minutes and then multiply by 60 we'll get the miles per hour he must average.

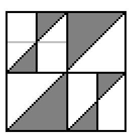
$$\frac{26\frac{1}{4}}{30} \times 60 = 26\frac{1}{4} \times 2 = 52\frac{1}{2} =$$

52.5 **Ans.**

TEAM ROUND

1. The positive integer divisors of 175, except 1, are arranged around a circle so that every pair of adjacent integers has a common factor greater than 1. We must find the sum of the two integers adjacent to 7.
First, determine the positive integer divisors of 175.
175 = 25 × 7 = 5 × 5 × 7. Thus the positive integer divisors of 175 (besides 1) are 5, 7, 25, and 35 and 175. Only two of these values are multiples of 7 and those are 35 and 175.
175 + 35 = 210 **Ans.**

2. The square quilt block is made up of four small congruent squares. The four small vertical rectangles in the block that are not squares are also congruent. We must find the ratio of the total shaded region to the quilt block.



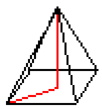
The area of the two larger triangles make up the area of one of the four small congruent rectangles. Each of the four small squares can themselves be broken up into 4 smaller squares (which I did in the upper left quadrant of the block). Each of the 4 smaller shaded triangles is $\frac{1}{2}$ of the area of one of the smaller squares, each of which is $\frac{1}{4}$ of the area of one of the four small congruent squares making it
$\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$ of the area of the block.
$\frac{1}{4} + \frac{4}{32} = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ **Ans.**

3. The ages of the 27 students signed up for the Spanish class are given below in the stem and leaf plot.
1 | 9 9
2 | 0 0 1 1 2 4 5 5 6 7 8 8
3 | 0 0 0 1 2 2 4 4 5 8 9
4 | 5 8
Let x = the mean
Let y = the median
Let z = the mode.
We must find the value of x(y-z).
The mode is just the most common value in the plot. 30 seems to be the only value that appears 3 times. No other value appears more so z = 30.
The median is just the middle value. There are 27 values so we need to find #14. That is 28. Therefore, y = 28.
Now for the mean:
19 + 19 = 38
(20 × 12) + 1 + 1 + 2 + 4 + 5 + 5 + 6 + 7 + 8 + 8 = 240 + 47 = 287
(30 × 11) + 1 + 2 + 2 + 4 + 4 + 5 + 8 + 9 = 330 + 35 = 365
45 + 48 = 93
38 + 287 + 365 + 93 = 783
$\frac{783}{27} = 29$
x = 29
x(y-z) = 29(28 – 30) = 29(-2) = -58 **Ans.**

4. If 40♦ represents a three-digit positive integer with a ones digit of ♦ and 1♦ is a two-digit positive integer with a ones digit of ♦, then what value of ♦ makes the equation
40♦ ÷ 27 = 1♦?
27 × 10 = 270
27 × 20 = 540
The value of ♦ is looking to be somewhere between 5 and 9 because we have to get the multiplication into the 400 range. Now what single digit multiplied by 7 results in a value whose one digit is that single digit? Certainly, 7 × 5 = 35
27 × 15 = 405
♦ = 5 **Ans.**

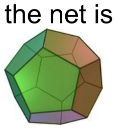
5. A right pyramid with a square base has all 8 edges of length 4 inches. What is the pyramid's volume?

The volume of a right pyramid is $\frac{1}{3}Bh$ where B is the area of the base.
B = 4 × 4 = 16
Now, what is h?
Looking at a picture of a right pyramid with a square base



we can see that the height is part of a right triangle one of whose sides is half the length of the diagonal of the square base and the other is a side of the pyramid all of whose sides are 4. The diagonal of the base is
$d^2 = 4^2 + 4^2 = 16 + 16 = 32$
$d = 4\sqrt{2}$
$\frac{1}{2}d = 2\sqrt{2}$
$h^2 + \left(2\sqrt{2}\right)^2 = 4^2$
$h^2 + 8 = 16$
$h^2 = 8$
$h = 2\sqrt{2}$
Substituting back into the volume equation, we have:
$V = \frac{1}{3} \times 16 \times 2\sqrt{2} = \frac{32\sqrt{2}}{3} \approx$
15.08494467… ≈ 15.08 **Ans.**

6. A dodecahedron, which is made up of 12 pentagons, looks like this when the net is put together.



Clearly you can see 5 vertices on the pentagon in front and, similarly, there are 5 vertices on the pentagon in the back. The other 10 pentagons are split into 5 which abut the front pentagon and 5 which abut the rear pentagon. Each of the first 5 share their vertices with the second 5. While each of the first 5 pentagons touch the second 5 pentagons at 3 vertices, one of them is shared with a neighbor. Thus, each of the 5 pentagons shares 2 vertices with the other 5.
5 × 2 = 10
10 + 10 = 20 vertices
For the edges, we have the 5 on the front pentagon and 5 on the rear pentagon. Then, there are 5 edges emanating from the vertices of the front pentagon and 5 emanating from the rear pentagon. Finally, there are 10 more where the two sets of 5 vertices meet each other.
10 + 10 + 10 = 30
20 + 30 = 50 **Ans.**

7. On the first day, the object's length increased by $\frac{1}{2}$. On the second day it further increased by $\frac{1}{3}$ and on the fourth day it further increased by $\frac{1}{4}$ and so on. On the nth day, the object has increased to 100 times its original size. We are asked to find n. Oh … I hope there's pattern here!

Day 1: $1 + \frac{1}{2} = \frac{3}{2}$

Day 2: $\frac{3}{2} + \left(\frac{1}{3} \times \frac{3}{2}\right) = \frac{3}{2} + \frac{1}{2} = 2$

Day 3: $2 + \left(\frac{1}{4} \times 2\right) = 2 + \frac{1}{2} = \frac{5}{2}$

Day 4: $\frac{5}{2} + \left(\frac{1}{5} \times \frac{5}{2}\right) = \frac{5}{2} + \frac{1}{2} = 3$

Whew! We have a pattern. Every two days we increase the length by 1!
From 3 times the length to 100 times the length will take 97 × 2 more days after this or 194.
194 + 4 = 198 **Ans.**

8. A cube is sliced with one straight slice which passes through two opposite edges. This is results in two solids. The area of the largest face on one of these two solids is $242\sqrt{2}$ square units. We are asked to find the exact surface area of the original cube. When you cut the cube in two like that, the largest face

of one of the cut pieces has sides of the edge of the cube and the diagonal an edge of the cube. If x is the edge of the cube then $x\sqrt{2}$ is the value of the diagonal.

$x \times x\sqrt{2} = 242\sqrt{2}$

$x^2 = 242$

The surface area of the original cube is $6x^2 = 6 \times 242 = 1452$ **Ans.**

9. Use the digits 2, 3, 4, 7 and 8 to form 5-digit positive integers. Only the digit 2 can be used more than once in any of the five-digit integers. So how many distinct five-digit integers can we make?
The number of five-digit integers where none of the digits are repeated is 5! = 120.
The number of different ways we can have a 5-digit number with 2 2's in it (i.e.,
22---, 2-2--, etc.) is
$\frac{5!}{2!3!} = \frac{5 \times 4}{2} = 10$ ways. For each way, we have to fill in the other three values. We have 4 choices for the first, 3 for the second and 2 for the last value.
4 × 3 × 2 = 24
24 × 10 = 240 5-digit numbers with 2 2's in it.
The number of different ways we can have a 5-digit number with 3 2's in it are:
$\frac{5!}{3!2!} = \frac{5 \times 4}{2} = 10$ ways.
But here we have 4 choices for the first non-2 value and 3 for the second non-2 value.
4 × 3 = 12
12 × 10 = 120 5-digit numbers with 3 2's in it.
The number of different ways we can have a 5-digit number with 4 2's in it are:
$\frac{5!}{4!1!} = 5$ ways
And for the single non-2 value, we have 4 choices.
5 × 4 = 20
And finally, there is only 1 way to have all 5 digits be 2 (22222).
120 + 240 + 120 + 200 + 1 = 501 **Ans.**

10. Ben places 42 bricks per hour. Bob places 36 bricks per hour. Bob worked twice as many hours as Ben and the two of them placed a total of 1254 bricks.
Let x be the number of hours that Ben works.
Let y be the number of hours that Bob works.
Then y = 2x
42x + (36 × 2x) = 1254
42x + 72x = 1254
114x = 1254
x = 11
Therefore, Ben worked 11 hours so he must have placed 11 × 42 = 462 bricks. 462 **Ans.**