2009
MATHCOUNTS CHAPTER

SPRINT ROUND

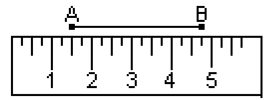
1. In the integer 45,075,123, by what factor would the value represented by the 5 in the thousands place have to be multiplied to equal the value represented by the 5 in the millions place?
The 5 in the thousands place represents 5000 or 5×10^3. The 5 in the millions place represents 5,000,000 or 5×10^6.
$\frac{5 \times 10^6}{5 \times 10^3} = 10^3$ = 1000 **Ans.**

2. Twenty-seven increased by twice a number is 39.
Let x be the number.
$27 + 2x = 39$
$2x = 12$
$x = 6$ **Ans.**

3. What is the positive difference between the number of students at the school with the largest enrollment and the number of students at the school with the smallest enrollment?
The number of students at the school with the largest enrollment is 1900.
The number of students at the school with the smallest enrollment is 1250.
1900 – 1250 = 650 **Ans.**

4. It takes June 4 minutes to ride her bike 1 mile. So how long does it take for her to go 3.5 miles?
Since it takes June 4 minutes to go 1 mile, it must take her
$4 \times 3.5 = 14$ minutes to go 3.5 miles.
14 **Ans.**

5. Two complete cycles of a pattern look like this:
AABBBCCCCCAABBBCCCCC…
Given that the pattern continues we are asked to find the 103rd letter.
How many characters are in a single pattern? AABBBCCCCC = 2 + 3 + 5 = 10 characters.
$\frac{103}{10} = 10R3$
Therefore, the 3rd element of the pattern is the answer. That is B. **Ans.**

6. What is the length of segment AB?


Point A is at 1.5.
Point B is at 4.75.
4.75 – 1.5 = 3.25 **Ans.**

7. What is the positive difference between $\frac{1}{2}+\frac{1}{3}$ and $\frac{1}{2}\times\frac{1}{3}$?
$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$
$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$
$\frac{5}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$ **Ans.**

8. 3 identical cylinders weigh as much as 5 spheres. 3 spheres weigh as much as 12 cubes. We are asked to find out how many cylinders weigh as much as 60 cubes.
Let c = cylinder
Let s = sphere
Let b = cube
$3c = 5s \rightarrow 9c = 15s$
$3s = 12b \rightarrow 15s = 60b$
$60b = 15s = 9c$
9 **Ans.**

9. Bill and Jill exercise on Monday, Jan. 1. Bill exercise every 5th day and Jill exercise every 4th day. The next day on which they both exercise is the LCM of 4 and 5.
$4 = 2 \times 2$
$5 = 5 \times 1$
The LCM is just $2 \times 2 \times 5 = 20$. Thus, in 20 days they will exercise together again. Twenty days is one day less than 3 weeks away, therefore, the next day they exercise together is a Sunday.
Ans.

10. The letters R, A, Y, N and A are on separate pieces of paper and placed in a bag. If one piece of paper is randomly selected what is the probability that it contains the letter R?

There are 5 pieces but only one R.

$\frac{1}{5}$ **<u>Ans.</u>**

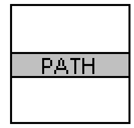
11. Amanda, Ben and Carlos share money in the ratio of 1:2:7, respectively. If Amanda's portion is $20, then what is the total amount of money shared?
From the ratio given, we know that Ben has twice what Amanda has, so he must have 2($20) = $40.
We also know that Carlos has 7 times what Amanda has, so he must have 7($20) = $140.
20 + 40 + 140 = 200 or 200.00 **<u>Ans.</u>**

12. If $\frac{x}{9} < \frac{2}{5}$ and x is a positive integer, how many distinct values are possible for x?
The LCM of 9 and 5 is 45. Turn both fractions into fractions with 45 as the denominator.
$\frac{5x}{45} < \frac{18}{45}$
$5x < 18$
The possible values are 1, 2 and 3. That is a total of 3 possible values. 3 **<u>Ans.</u>**

13. Louis earns $1200 per month with 5% commission on sales. He sold $25,000 during the month. His commission was
$0.05 \times 25{,}000 = 1250$
But don't forget his base salary.
1200 + 1250 = 2450 or 2450.00 **<u>Ans.</u>**

14. Set L has four consecutive, positive, odd integers. The sum of the greatest integer and twice the least integer is 39.
We are asked to find the least integer in the set.
Let x be the least integer in the set.
Then the other 3 integers are $x + 2$, $x + 4$ and $x + 6$.
$(x + 6) + 2x = 39$
$3x + 6 = 39 \rightarrow 3x = 33$
$x = 11$ **<u>Ans.</u>**

15. We have a square garden with an area of 1600. A straight path will run from the middle of one side to the middle of the opposite side. The path is 3 feet wide so how many square feet of garden will be turned into path?



Since the area of a square = s^2, each side of the garden is $\sqrt{1600} = 40$ feet.
The path is 3 feet wide so the grey area, which represents the path, is 40 feet wide by 3 feet long.
$40 \times 3 = 120$ **<u>Ans.</u>**

16. For integers a, b and k, $a > 12$, $b < 20$ and $a < b$. If $b = 7k$, then what is the value of k?
If b is less than 20, k can be 0, 1, or 2, making b 0, 7 or 14. We don't need to worry about negative integers because $a > 12$ and $a < b$ which means that $12 < b < 20$. Considering this fact, we see that the only possible answer for b is 14.
Therefore, $k = 2$. **<u>Ans.</u>**

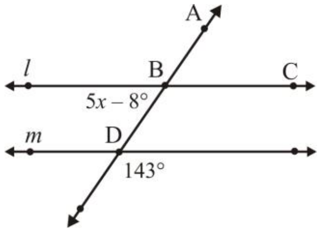
17. $\frac{a}{b} = \frac{5}{9}$
$\frac{a}{b+5} = \frac{1}{2}$
We are asked to find $a + b$.
$9a = 5b$
$2a = b + 5 \rightarrow b = 2a - 5$
$9a = 5(2a - 5)$
$9a = 10a - 25 \rightarrow a = 25$
$b = 2a - 5 = b = 2(25) - 5 = 50 - 5 = 45$
$a + b = 25 + 45 = 70$ **<u>Ans.</u>**

18. How many positive whole number divisors does 196 have?
To find the answer starting with the number 1, list sets of 2 integers whose product is 196.

1	196
2	98
4	49
7	28
14	14

If we go any higher we'll just be repeating ourselves. There are 5 sets of 2 but one of those (14) is repeated twice.
10 – 1 = 9 **<u>Ans.</u>**

19. Lines l and m are parallel and points A, B, and D are collinear. We are asked to find the value of *x*.

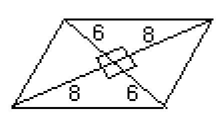


The angle at D is congruent to the 143 degree angle. The angle with the measure 5*x* – 8 is supplementary to angle D.
$5x - 8 + 143 = 180$
$5x + 135 = 180$
$5x = 45$
$x = 9$ **Ans.**

20. What is the only integer value of *n* for which $\frac{n+1}{13-n}$ is a positive prime number?
How do we go about this? Well, a positive prime means that both the denominator and numerator are positive or negative. Also, we're looking for an integer value, which means that $n + 1$ is a multiple of $13 - n$ and $n + 1 > 13 - n$. At what point does that happen?
Well, what happens if *n* is 1?
Then the fraction evaluates to $\frac{2}{11}$.
Looks like we have to go to about halfway between 1 and 13 to get a fraction that might evaluate to an integer. Try $n = 6$ and the fraction is $\frac{6}{6} = 1$ Are we there? No, because 1 is not a prime number. So how far do we need to look? Well, as soon as we have $n = 14$, the denominator will be negative and the numerator positive. And if $n = 13$, the denominator will be 0, which doesn't work! Therefore, we only have to look at the fractions where $6 < n < 13$.
$n = 7$; fraction is $\frac{8}{6}$. No
$n = 8$; fraction is $\frac{9}{5}$
Are you seeing a pattern here?
$\frac{8}{6}, \frac{9}{5}, \frac{10}{4}, \frac{11}{3}, \frac{12}{2}, \frac{13}{1}$
Do any of these fractions resolve to integers? Yes, the last two but the first of those resolves to 6 which is not a prime. The second resolves to 13 which is a prime. Therefore $n = 12$ **Ans.**

21. A set of 7 positive integers has a unique mode of 1. It also has a mean of 5 and a median of 6. We are asked to find the largest possible value for any of the integers in the set.
7 numbers and a mean of 5 means that the sum of all 7 numbers is $7 \times 5 = 35$.
1 is the mode so we know that there must be at least two 1s. The median is 6 which means that the 4^{th} value is 6.
Since we are looking for the largest possible value that can be in the set, we want the other numbers to be as small as possible. 1 is the smallest positive integer so let's assume that we have three 1s.
1, 1, 1, 6, *x*, *y*, *z*
The smallest possible value for *x* would also be 6 since that would not stop the median from being 6 or the mode from being 1.
1, 1, 1, 6, 6, y, *z*
y cannot be 6 because 1 would no longer be the only mode. So let *y* be 7.
1, 1, 1, 6, 6, 7, *z*
$z + 1 + 1 + 1 + 6 + 6 + 7 = 35$
$z + 22 = 35$
$z = 13$ **Ans.**

22. The diagonals of a rhombus are perpendicular bisectors of each other. The lengths of the diagonals are 12 and 16. We are asked to find the perimeter of the rhombus.



By entering the given measures, we can see that this has created four 6 × 8 × 10 right triangles. Therefore, the side of the rhombus is 10. $10 \times 4 = 40$ **Ans.**

23. Chun can score 0, 2, 6 or 10 points in each round of a game. After four rounds the sum of his scores is 16. How many

different scoring sequences could produce 16?
Since 16 is not a possible score for one round we must start by looking at whether 16 is a possible score for 2 rounds.
It is. 10, 6, 0, 0
What about in 3 rounds?
There is no way. So we have only to look at taking all 4 rounds to get to 16.
10, 2, 2, 2
6, 6, 2, 2
That's it. Now we need to find how many different orders of each scenario we can have.
The number of different sequences using 10, 6, 0, 0 is:
$\frac{4!}{2!} = \frac{24}{2} = 12$
The number of different sequences using 10, 2, 2, 2 is:
$\frac{4!}{3!} = 4$
The number of different sequences using 6, 6, 2, 2 is:
$\frac{4!}{2!2!} = \frac{24}{4} = 6$
12 + 4 + 6 = 22 **Ans.**

24. Janice bought 30 items. They cost either 30 cents, 2 dollars or 3 dollars. She spent $30 and we must find how many 30-cent items she bought.
Let x = the number of 30-cent items
Let y = the number of 2-dollar items
Let z = the number of 3-dollar items
$x + y + z = 30$
$0.3x + 2y + 3z = 30$
We have 2 equations and 3 unknowns. That presents a problem. Let's try and cut this down. First of all, the 30-cent items must be purchased in a multiple of 10 units simply because that's the only way we'll get a value in dollars with no cents. The other two items are in terms of dollars. So there are either 10 30-cent items or 20 30-cent items. (It can't be 30 30-cent items because that only costs $9.)
So let's try 20 items. Then $6 is spent and we get:
$20 + y + z = 30$
$y + z = 10 \rightarrow z = 10 - y$
$6 + 2y + 3z = 30$
$2y + 3z = 24$
$2y + 3(10 - y) = 24$
$2y + 30 - 3y = 24$
$2y - 3y = -6$
$y = 6$
Does this work?
20 + 6 + 4 = 30
6 + 12 + 12 = 30
Yes it does. But we should check $x = 10$ just to be safe.
If we only bought 10 30-cent items it would cost $3. Then we get:
$10 + y + z = 30$
$y + z = 20 \rightarrow z = 20 - y$
$3 + 2y + 3z = 30$
$2y + 3z = 27$
$2y + 3(20 - y) = 27$
$2y + 60 - 3y = 27$
$2y - 3y = -33$
$y = 33$ (Which is too many items.)
Therefore, $x = 20$. **Ans.**

25. The ratio of $2x - y$ to $x + y$ is 2 to 3. We are asked to find the ratio of x to y.
$\frac{2x - y}{x + y} = \frac{2}{3}$
$6x - 3y = 2x + 2y$
$4x = 5y$
$\frac{x}{y} = \frac{5}{4}$ **Ans.**

26. If the members of a band are arranged in 8 rows, there are 2 positions unoccupied in the formation. If they are arranged in 9 rows, there are 3 positions unoccupied. We are asked to find how many members are in the band if the membership is between 100 and 200.
The fact that using 8 rows leaves 2 positions unfilled says that the number of members in the band is 2 less than a multiple of 8. That makes this an even number. We also know that the number of members is 3 less than a multiple of 9. Let's just enumerate even numbers that are 3 less than multiples of 9 between 100 and 200 and are even.
117 – 3, 135 – 3, 153 – 3, 171 – 3, 189 – 3 or 114, 132, 150, 168, 186
Now the numbers that are two less than a multiple of 8:
102, 110, 118, 126, 134, 142, 150, 158, 166, 174, 182, 190, 198
150 is in both lists. Does it satisfy the requirements?

$\frac{150}{8} = 18R6$ (8 – 6 = 2)

$\frac{150}{9} = 16R6$ (9 – 6 = 3)

Yes it does. 150 **Ans.**

27. Jerry is twice as old as his brother. He is 6 years older than his sister. We are asked to find in how many years will Jerry's will be $\frac{2}{3}$ of the combined ages of his brother and sister.

Let J = Jerry's age
Let S = Jerry's sister's age
Let B = Jerry's brother's age

J = 2B → B = $\frac{J}{2}$

J = S + 6 → S = J – 6

Let x = the number of years that must pass before Jerry's age is $\frac{2}{3}$ of the combined ages of his brother and sister.

$J + x = \frac{2}{3}(B + x + S + x)$

$3J + 3x = 2B + 2x + 2S + 2x$

$3J + 3x = 2(\frac{J}{2}) + 2x + 2(J - 6) + 2x$

$3J + 3x = J + 2x + 2J - 12 + 2x$

$3J + 3x = 3J + 4x - 12$

$3x = 4x - 12$

$x = 12$ **Ans.**

28. In a sequence of positive integers each term after the first is $\frac{1}{3}$ of the sum of the term that precedes it and the term that follows it. The first term is 2 and the 4th is 34 and we are asked to find the 5th term.

Let x = the second term
Let y = the third term
Let z = the fifth term

$x = \frac{1}{3}(2 + y)$ → $3x = 2 + y$ → $y = 3x - 2$

$y = \frac{1}{3}(x + 34)$

$3x - 2 = \frac{1}{3}(x + 34)$ → $9x - 6 = x + 34$

$8x = 40$

$x = 5$

$y = 3x - 2 = 15 - 2 = 13$

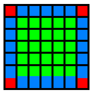
So the first 4 terms in the series are:
2, 5, 13, 34

$34 = \frac{1}{3}(13 + z)$

$102 = 13 + z$

$z = 89$ **Ans.**

29. We have a 7-by-7 checkerboard.



We are asked to find the probability that two squares chosen at random, without replacement, are adjacent to each other. Corner squares (marked in red) have only 2 adjacent squares.

There are a total of 49 squares. So the probability that we pick one of the corner (red) squares and then one of its adjacent squares is:

$\frac{4}{49} \times \frac{2}{48} = \frac{8}{2352}$

All the rest of the edge squares (in blue) have 3 adjacent squares. There are 5 × 4 = 20 edge squares. So the probability that we pick one of the blue (edge) squares and then one of its adjacent squares is:

$\frac{20}{49} \times \frac{3}{48} = \frac{60}{2352}$

Finally, each of the internal (green) squares has 4 adjacent squares. There are 5 × 5 = 25 green squares. So the probability that we pick one of the internal (green) squares and then one of its adjacent squares is:

$\frac{25}{49} \times \frac{4}{48} = \frac{100}{2352}$

$\frac{8}{2352} + \frac{60}{2352} + \frac{100}{2352} = \frac{168}{2352} = \frac{21}{294} =$

$\frac{7}{98} = \frac{1}{14}$ **Ans.**

30. The equation for a circle is:
$(x - h)^2 + (y - k)^2 = r^2$ where r is the radius and h and k are the x- and y-coordinates, respectively, of the center of the circle.

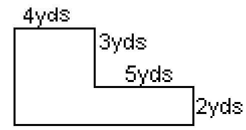
We are told that the center of the circle is (5, 15) and the radius is $\sqrt{130}$. So the equation becomes:

$(x - 5)^2 + (y - 15)^2 = \sqrt{130}^2 = 130$
Point Q(x, y) is on the circle and has integer coordinates. The x-coordinate of point Q is twice the value of the y-coordinate and we are asked to find the maximum possible value of x.
So $x = 2y$.
Substituting into the equation:
$(2y - 5)^2 + (y - 15)^2 = 130$
$4y^2 - 20y + 25 + y^2 - 30y + 225 = 130$
$5y^2 - 50y + 250 = 130$
$5y^2 - 50y + 120 = 0$
$y^2 - 10y + 24 = 0$
$(y - 6) \times (y - 4) = 0$
$y = 6, y = 4$
Since we are asked to find the maximum possible value of x, then y must also be as big as possible.
Therefore, $y = 6$
$x = 2y = 12$ **Ans.**

TARGET ROUND

1. What fraction is exactly half-way between $\frac{2}{3}$ and $\frac{4}{5}$?
   The GCF of 3 and 5 is 15 so let's rewrite the two fractions as:
   $\frac{10}{15}$ and $\frac{12}{15}$
   Thus, the half-way point is $\frac{11}{15}$ **Ans.**

2. We have a stem and leaf plot:
   4| 9
   5| 2 3 5 8 8 9
   6| 0 1 1 2 6 8 9 9
   These represent heights and we are asked to find the mean height of the players on the team.
   Let's add up all the heights.
   There is only one value in the 40's but let's find sum of the 6 values we have in the 50's.
   52 + 53 + 55 + 58 + 58 + 59 = 335
   We have 8 values in the 60's. Let's add those up.
   60 + 61 + 61 + 62 + 66 + 68 + 69 + 69 = 516
   Total them up:
   49 + 335 + 516 = 900
   Now, how many players do we have?
   1 + 6 + 8 = 15
   $\frac{900}{15} = 60$ **Ans.**

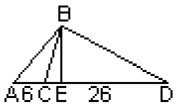
3. We have to carpet the region shown in the diagram.
   

   The carpet costs $21.95 per square yard and the padding costs $2.55 per square yard.
   We can divide the region into two rectangular regions to make computing easier. The first is 4 × 3 and the second is (4 + 5) × 2.
   4 × 3 = 12
   (4 + 5) × 2 = 18
   12 + 18 = 30
   So we have a total of 30 sq yards.
   30 × (21.95 + 2.55) =
   30 × 24.50 = 735 or 735.00 **Ans.**

4. 8 lb. of feathers and 2 oz. of gold cost $932.
   14 lb. of feathers and 3 oz. of gold cost $1402.
   We are asked to find the cost of 5 lb. of feathers and 5 oz. of gold.
   Let f = the cost of a pound of feathers
   Let g = the cost of an ounce of gold
   $8f + 2g = 932 \rightarrow 24 + 6g = 2796$
   $14f + 3g = 1402 \rightarrow 28f + 6g = 2804$
   $4f = 8$
   $f = 2$
   $8f + 2g = 932 \rightarrow 8(2) + 2g = 932 \rightarrow$
   $16 + 2g = 932 \rightarrow 2g = 916 \rightarrow g = 458$
   $5f = 5(2) = 10$
   $5g = 5(458) = 2290$
   2290 + 10 = 2300 or 2300.00 **Ans.**

5. What is the probability of getting a sum larger than 10 when you roll 2 standard 6-sided dice?
   We must find the probability of getting either 11 or 12.
   For 11 we could roll either 6 and 5 or 5 and 6.
   For 12 we could roll only 6 and 6.
   $\frac{3}{6 \times 6} = \frac{3}{36} = \frac{1}{12}$ **Ans.**

6. In the diagram, the area of triangle ABC is 27 square units. We are asked to find the area of triangle BCD.



Drop a perpendicular from B to side AD, label the point of intersection E. BE is the height of triangle ABC (as well as the height of triangles ABD and BCD).
Let BE = h

$\frac{1}{2} \times h \times 6 = 27$

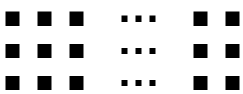
$3h = 27$

$h = 9$

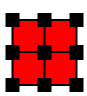
The area of triangle BCD is:

$\frac{1}{2} \times 9 \times 26 = 9 \times 13 = 117$ **Ans.**

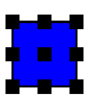
7. There is a total of 70 squares of 3 sizes whose vertices are points on this rectangular $3 \times n$ grid. So what is the value of n?



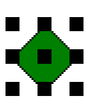
Let's look at a 3×3 grid, a 3×4 grid and a 3×5 grid. With any luck we should see a pattern.
3×3 grid: There are 4 unit squares.



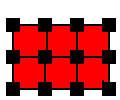
There is just 1 square of area 4.



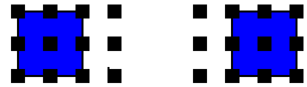
And there is just 1 square of area 2.



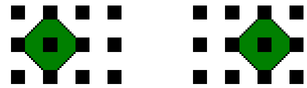
Now let's look at a 3×4 grid.
There are 6 unit squares.



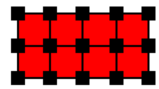
There are 2 squares of area 4.



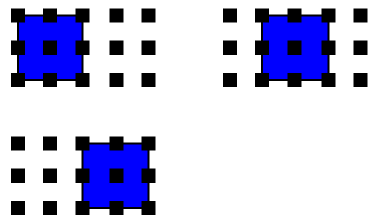
There are 2 squares of area 2.



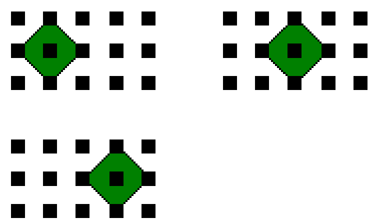
Finally, let's look at a 3×5 grid.
There are 8 unit squares.



There are 3 squares of area 4.



There are 3 squares of area 2.



Do we have a pattern for each of these?
Yes, we do.
Let's look at unit squares first.
3×3: 4
3×4: 6
3×5: 8
$3 \times n$: $2 + 2(n - 2)$
Now let's look at squares of area 4.
3×3: 1
3×4: 2
3×5: 3
$3 \times n$: $n - 2$
And squares of area 2 are exactly like squares of area 4.
$2 + 2(n - 2) + (n - 2) + (n - 2) = 70$
$2 + 2n - 4 + 2n - 4 = 70$
$4n - 6 = 70$
$4n = 76$
$n = 19$ **Ans.**

8. The fraction $\frac{x^5}{3^x}$ is equal to a positive integer and x is an integer greater than 4. We are asked to find *x*.
3^x is a set of 3's all multiplied. If the fraction is to resolve to an integer, then *x* must be a multiple of 3. Since x is an integer greater than 4, let's try 6.
$\frac{6^5}{3^6} = \frac{6\times6\times6\times6\times6}{3\times3\times3\times3\times3\times3} = \frac{2\times2\times2\times2\times2}{3}$
Well, that's not going to do it. Now we can see that x should not only be a multiple of 3, it should be a power of 3. Let's try 9.
$\frac{9^5}{3^9} = \frac{3^{10}}{3^9} = 3$
x = 9 **Ans.**

TEAM ROUND

1. 10 pictures, 4 inches by 6 inches each are mounted on a poster board which measures 20 inches by 17 inches. We are asked to find out how much of the poster background will show.
The area of the poster board is
20 × 17 = 340
The area covered by the 10 pictures is
10 × 4 × 6 = 240
340 – 240 = 100 **Ans.**

2. Let *n* = the number of sides in a regular polygon where 3 ≤ *n* ≤ 10. We are asked to find how many values of n result in a regular polygon where the common degree measure of the interior angles is non-integral.
The sum of the interior angles of a polygon is 180(*n* – 2).
We can dispense with *n* = 3 or *n* = 4. We **know** those are 60 and 90.

n	sum	angle
5	540	108
6	720	120
7	900	**128.571**
8	1080	135
9	1260	140
10	1440	144

Only 1 n-value gives non-integer angle measures and that's *n* = 7.
1 **Ans.**

3. 5 players all scored a different number of points in the game. Each player scored more than 9 points. Amanda scored the fewest and Kara scored the second fewest. Kara scored 16 points. The average of the 5 players is also 16. We are asked to find how many points Amanda scored.
The players scored a total of 16 × 5 = 80 points. Subtract Kara's score and you have 80 – 16 = 64. Since everyone scored a different number of points the three players who scored higher than Kara must have scored something like 17, 18 and 19. Will this work?
17 + 18 + 19 = 54
54 + 16 = 70
80 – 70 = 10
This works but could one of the higher scoring players scored even more? What if the player who scored 19 actually scored 20?
17 + 18 + 20 = 55
55 + 16 = 71
80 – 71 = 9 but we were told that each player scored more than 9 points.
10 **Ans.**

4. The ratio of domestic stamps to foreign stamps is 3:1. If John sells 30 of his domestic stamps, then the ratio of domestic stamps to foreign stamps is 1:2. So how many foreign stamps does John have?
Let *d* = the number of domestic stamps
Let *f* = the number of foreign stamps
Then:
$\frac{d}{f} = \frac{3}{1} \rightarrow d = 3f$
If John makes the sale then:
$\frac{d-30}{f} = \frac{1}{2} \rightarrow 2d - 60 = f \rightarrow$
$d = 3(2d - 60) \rightarrow d = 6d - 180 \rightarrow$
$5d = 180 \rightarrow d = 36$
$36 = 3f$
$f = 12$ **Ans.**

5. What is the 200^{th} term of the increasing sequence of positive integers formed by omitting only the perfect squares?
Let's enumerate the number of perfect squares ≤ 200.
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196
That's 14 perfect squares.

So if we removed them from the first 200 numbers we'd be left with 186 numbers. What's the next perfect square?
It's 225. That's too far.
200 + 14 = 214 **Ans.**

6. Keisha has 45 coins. She has one fewer nickel than she has dimes. She has one fewer quarter than 3 times the number of nickels. So how many nickels does she have?
   Let n = the number of nickels
   Let d = the number of dimes
   Let q = the number of quarters
   $n + d + q = 45$
   $d = n + 1$
   $q = 3n - 1$
   $n + n + 1 + 3n - 1 = 45$
   $5n = 45$
   $n = 9$ **Ans.**

7. $6^{x+1} - 6^x = 1080 = 6^3 \times 5$
   $6^x(6 - 1) = 6^3 \times 5$
   $6^x \times 5 = 6^3 \times 5$
   $6^x = 6^3$
   x = 3 **Ans.**

8. The mean of 3 numbers is $\frac{5}{9}$.
   The difference between the largest and smallest number is $\frac{1}{2}$.
   $\frac{1}{2}$ is one of the numbers.
   We must find the smallest number.
   The sum of the 3 numbers is
   $3 \times \frac{5}{9} = \frac{15}{9}$
   Since $\frac{1}{2}$ is one of the numbers the other two must sum to
   $\frac{15}{9} - \frac{1}{2} = \frac{30}{18} - \frac{9}{18} = \frac{21}{18} = \frac{7}{6}$
   Let x = the first number.
   Let y = the second number.
   $x + y = \frac{7}{6}$
   $x - y = \frac{1}{2} = \frac{3}{6} \rightarrow y = x - \frac{3}{6}$
   $x + (x - \frac{3}{6}) = \frac{7}{6} \rightarrow 2x = \frac{7}{6} + \frac{3}{6} = \frac{10}{6}$
   $x = \frac{5}{6}$
   $y = \frac{7}{6} - \frac{5}{6} = \frac{2}{6} = \frac{1}{3}$ **Ans.**

9. $w^3 + x^3 + y^3 = z^3$
   w^3, x^3, y^3 and z^3 are distinct, consecutive positive perfect cubes listed in ascending order. We are asked to find the smallest possible value of z.
   Well, that's what we have a calculator for I guess…
   $1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36 \neq 4^3$
   $2^3 + 3^3 + 4^3 = 8 + 27 + 64 = 99 \neq 5^3$
   $3^3 + 4^3 + 5^3 = 26 + 64 + 125 = 216 = 6^3$
   $z = 6$ **Ans.**

10. A bag contains red, white, green and blue marbles. There are an equal number of red and white marbles. There are 5 times as many green marbles as blue marbles. There is a 35% change of selecting a red marble first. We are asked to find the fewest possible number of green marbles in the bag.
    Let r = the number of red marbles
    Let w = the number of white marbles
    Let g = the number of green marbles
    Let b = the number of blue marbles.
    $r = w$ and $g = 5b$
    Since $r = w$, there is also a 35% chance of selecting a white marble first. That means there is only a
    100 – (35 + 35) = 30% chance of selecting a green or blue. But we know that there are 5 times as many green marbles as blue so there is a 25% chance of selecting a green marble and 5% chance of selecting a blue marble.
    35% of any number less than 20 will not result in an integer so let's look at whether 20 total marbles works.
    $r = 7$; $w = 7$; $g = 5$; $b = 1$
    Yes. 5 **Ans.**