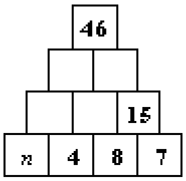
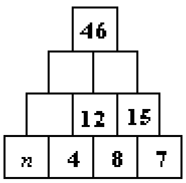
2010
MATHCOUNTS CHAPTER

SPRINT ROUND

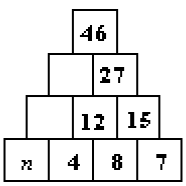
1. We are given a Number Wall, where you add the numbers next to each other and write the sum in the block directly above the two numbers. We are asked to find the value of ‘*n*’.



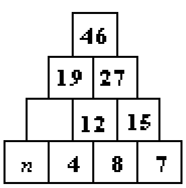
First we can add 4 and 8 to get 12.



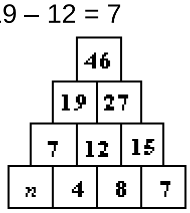
Next we can add 12 and 15 to get 27.



We no longer have two numbers adjacent to each other to add. But we do have 46 and 27. We can subtract 27 from 46 to fill in the box at the left of 27.
46 – 27 = 19



And we can subtract 12 from 19 to fill in the box at the left of 12.
19 – 12 = 7



Finally, we can solve for *n*.
$n + 4 = 7$
$n = 3$ **Ans.**

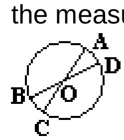
2. We are asked to find how many integers between 500 and 1000 contain both the digits 3 and 4.
Between 500 and 600 we have 534 and 543 or 2 numbers. Similarly, for 600-700, 700-800, 800-900 and 900-1000 there are 2 numbers containing 3 and 4 in each 100 number range.
2 × 5 = 10 **Ans.**

3. Blue, green and white paint are combined in the ratio 3:2:4, respectively. 12 quarts of white paint are used and we are asked to find how many quarts of green paint are needed.
The ratio of green to white paint is 2:4, thus, the amount of green paint is always 1/2 the amount of white paint.
(1/2) × 12 = 6 **Ans.**

4. $m \blacklozenge n = \left(m^2 - n\right) \div n$, for all real numbers *m* and *n*, where $n \neq 0$.
$6 \blacklozenge 3 = \left(6^2 - 3\right) \div 3 = \left(36 - 3\right) \div 3 =$
$\frac{33}{3} = 11$ **Ans.**

5. How many six-inch by six-inch square tiles are needed to cover a three-foot by two-foot rectangular floor?
First change six inches into feet. What we really have are $\frac{1}{2}$-foot × $\frac{1}{2}$-foot tiles. Each occupies an area of $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ sq. feet. We also know that we can fit an integral number of tiles on the floor and not have any space left over. Therefore, we only need divide the area of the floor by the area of a tile.
The area of the floor is 3 × 2 = 6.
$\frac{6}{\frac{1}{4}} = 6 \times 4 = 24$ **Ans.**

6. We are given a circle with center O and diameters AC and BD. The angle AOD measures 54º and we are asked to find the measure of angle AOB.



Angles AOD and AOB are

supplementary.
180 – 54 = 126 **Ans.**

7. A copy of the Times costs \$0.95 more than a copy of the News. Both papers together cost \$7.25. We are asked to find the price of the News.
Let x = the price of the Times.
Let y = the price of the News. Now we can say that:
$x = y + 0.95 \rightarrow x - y = 0.95$
and
$x + y = 7.25$
Subtract the first equation from the second.
$2y = 6.30$
$y = 3.15$ **Ans.**

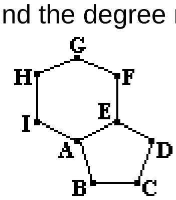
8. We are asked to find the value of m where the graphs of the three lines
$y = 2x + 1$
$x = 3$
$y = mx + 3$
have a single point of intersection.
This happens when the value of x and y are the same for all 3 equations. Since one of the equations is $x = 3$, we know that x will have to be 3 and we can plug it into the first equation to get our y-value.
$y = (2 \times 3) + 1 = 6 + 1 = 7$
Now we can plug our x- and y-values into the third equation and solve for m.
$7 = (m \times 3) + 3$
$7 = 3m + 3$
$3m = 7 - 3 = 4$
$m = \frac{4}{3}$ **Ans.**

9. 20 leaves of a 40-page book are numbered 1 through 40. We are asked to find the sum of the pages in the book after we remove the first and last leaves.
Let's think about this. If you had a book of just 1 page the front of the page would be numbered 1 and the back 2. And if you had a book of 3 pages the first page would be as before but the last would be numbered 5 and 6. In this case we have a 40-page book so the first page has numbers 1 and 2 and the last page has numbers 39 and 40.
Let's start by adding up the numbers 1 through 40.
1 + 2 + … + 39 + 40 =
(1 + 40) + (39 + 2) + … (20 + 21) =
20 × 41 = 820
Now let's find the sum of the page numbers we remove.
1 + 2 + 39 + 40 = (1 + 40) + (39 + 2) =
41 × 2 = 82
This leaves 820 – 82 = 738 **Ans.**

10. John ate three-fourths of the jelly beans in a bag. That means that there is now just one-fourth of the bag left.
Mike ate two-thirds of the remaining jelly beans. So Mike ate:
$\frac{2}{3} \times \frac{1}{4} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of the bag.
That means that
$\frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$ of the bag is left.
We are told that Fred ate the last 10 jelly beans which comprise $\frac{1}{12}$ of the bag.
That means that there are 12 × 10 = 120 jelly beans in the whole bag.
120 **Ans.**

11. Joshua took 4 tests and got a median score of 80. The difference between his greatest score and his least score is 12. We are asked to find the maximum possible value of the mean of his scores.
Since there is an even number of tests, the median is the average of the middle two scores. Let's assume that both are 80 since we're looking for the greatest mean and we want the lowest test score to be as high as possible.
Then we have x, 80, 80, y as the 4 test scores. Since the difference between the greatest and least score is 12 we should set x = 80 (again, since we're looking for the greatest mean and we want the lowest test score to be as high as possible.) and y = 92.
So for the mean we get:
$\frac{80+80+80+92}{4} = \frac{332}{4} = 83$ **Ans.**

12. Regular pentagon ABCDE and regular hexagon AEFGHI are drawn on opposite sides of line segment AE such that they are coplanar. We are asked to find the degree measure of angle DEF.



The sum of the angles of a regular polygon with n sides is given by the formula: $180 \times (n - 2)$
For the pentagon, we have 5 sides, so the sum of the angles in the pentagon is
$180 \times (5 - 2) = 180 \times 3 = 540$
This means that each angle in the pentagon is:
$\frac{540}{5} = 108^o$
In particular, angle AED is 108º.
The sum of the angles in a regular hexagon is
$180 \times (6 - 2) = 180 \times 4 = 720$
This means that each angle in the hexagon is:
$\frac{720}{6} = 120^o$
In particular, angle AEF is 120º.
The sum of angles AEF, AED and DEF is 360º.
Let x = the degree measure of angle DEF.
$108 + 120 + x = 360$
$228 + x = 360$
$x = 132$ **Ans.**

13. Let x = the first number.
Let y = the second number.
The sum of the two numbers is 20.
$x + y = 20$
Four times the larger number exceeds three times the smaller number by 143.
Let's assume that x is the larger number.
$4x = 3y + 143$
Since $x + y = 20$, we know $y = 20 - x$ and we can use substitution to solve.
$4x = 3(20 - x) + 143$
$4x = 60 - 3x + 143$
$4x = 203 - 3x$
$7x = 203$
$x = 29$ **Ans.**

14. The first and seventh terms of a sequence are each 10. Starting with the third term, each term is the sum of the previous two terms, so what is the fifth term?
Let x_1 through x_7 represent the terms.
$x_1 = 10$ and $x_7 = 10$
Let's write out the sequence.
$10, x_2, x_3, x_4, x_5, x_6, 10$
Since each term, starting with the third term is the sum of the previous two, we can rewrite the sequence as:
$10, x_2, 10 + x_2, 10 + 2x_2, 20 + 3x_2, 30 + 5x_2, 50 + 8x_2$
But $50 + 8x_2$ is the seventh term so it is equal to 10.
$50 + 8x_2 = 10$
$8x_2 = -40$
$x_2 = -5$
The fifth term is $20 + 3x_2$, so substituting -5 for x_2 gives us $20 + (3 \times -5) =$
$20 - 15 = 5$ **Ans.**

15. We create four-digit whole numbers formed only with the digits 1, 3, 5 and 7. Repetition of digits is allowed. We are asked to find the probability that one of these four-digit numbers is a palindrome.
A palindrome reads the same backwards as forwards so 1111 is a palindrome, as are 1331, 1551 and 1771.So there are 4 palindromes starting with 1. Similarly there are 4 palindromes starting with 3, 5 and 7 respectively for a total of 16 palindromes. 16 **Ans.**

16. The time required to pave a road is directly proportional to the length of the road and inversely proportional to the number of workers on the job. We need to find out how many days it would take for 64 people to pave 2 miles if it takes 96 people 6 days to pave 1 mile.
96 people for 6 days means it takes 96×6 people-days to pave 1 mile. To pave 2 miles means it takes $96 \times 6 \times 2$ people-days. But we have only 64 people. So it takes
$\frac{96 \times 6 \times 2}{64} = \frac{3 \times 6 \times 2}{2} = 18$ days to pave 2 miles. 18 **Ans.**

17. At 6:00 a clock chimes 6 times with the final chime beginning exactly 10 seconds after 6:00. Given that the chimes are uniformly spaced, how long after 12:00 does the twelfth chime begin?
First, let's find out the time between chimes. Forget about the first one since it starts at 6:00 on the dot. That leaves us 5 chimes to get in with the last ending 10 seconds later. That says that the interval between chimes is:
$\frac{10}{5} = 2$ seconds.
So, at 12:00 the first chime starts. That leaves 11 more.

$11 \times 2 = 22$ **Ans.**

18. A bag has 6 balls, two red and four green. We are asked to find the probability that two selected balls (without replacement) are of different colors. Let's find the probability of picking a red ball and then a green ball.
$\frac{2}{6} \times \frac{4}{5} = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$
The probability of picking a green ball and then a red ball is
$\frac{4}{6} \times \frac{2}{5} = \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$
Thus, the total probability is
$\frac{4}{15} + \frac{4}{15} = \frac{8}{15}$ **Ans.**

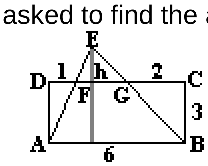
19. What is the value of
$\left(1 - \frac{1}{2}\right) + \left(2 - \frac{1}{2}\right) + \left(3 - \frac{1}{2}\right) + \ldots$
$\left(50 - \frac{1}{2}\right)$?
$\left(1 - \frac{1}{2}\right) + \left(2 - \frac{1}{2}\right) + \left(3 - \frac{1}{2}\right) + \ldots$
$\left(50 - \frac{1}{2}\right) =$
$\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \ldots + \frac{99}{2} =$
$\frac{(1+99) + (3+97) + \ldots + (49+51)}{2} =$
$\frac{25 \times 100}{2} = 25 \times 50 = 1250$ **Ans.**

20. How many subsets of {1, 2, 3, 4, 5, 6} have either 4 or 5 as their largest element?
To figure this one out we can look at how many total subsets there are in {1, 2, 3, 4, 5} (notice we can leave 6 out because any of the sets containing 6 won't have 4 or 5 as the largest element) and subtract from that the number of subsets that do NOT have 4 or 5, ie. all of the subsets from the set {1, 2, 3}. The number of subsets can be calculated by 2^n, where *n* is the number of elements in the set. Thus, the number of subsets possible from {1, 2, 3, 4, 5} is $2^5 = 32$ and the number of subsets possible from the set {1, 2, 3} is $2^3 = 8$.
$32 - 8 = 24$ **Ans.**

21. The average of A and 3B is 7, i.e.,
$\frac{A + 3B}{2} = 7$
The average of A and 3C is 8.
$\frac{A + 3C}{2} = 8$
The average of A and 3D is 9.
$\frac{A + 3D}{2} = 9$
We are asked to find the average of A, B, C and D.
A + 3B = 14
A + 3C = 16
A + 3D = 18
Add all three equations together.
3A + 3B + 3C + 3D = 14 + 16 + 18 = 48
A + B + C + D = 16
Thus, the average of A, B, C and D is
$\frac{A + B + C + D}{4} = \frac{16}{4} = 4$ **Ans.**

22. Ms. Quinn traveled at a constant speed for a distance of 50 miles. Later she traveled at a constant speed for a distance of 400 miles but this time she drove 3 times as fast. We are asked to find the ratio of the time it took for her second trip to the time it took for her first trip.
If Ms. Quinn travels at a speed of *x* miles per hour for the first trip, it takes her $\frac{50}{x}$ hours to complete the first trip.
The second trip is done three times as fast so she travels at a speed of 3*x* miles per hour. Therefore, for the second trip, it takes Ms. Quinn $\frac{400}{3x}$ hours to travel the distance.
The ratio of the time for the second trip to the time for the first trip is:
$\frac{\frac{400}{3x}}{\frac{50}{x}} = \frac{\frac{400}{3}}{50} = \frac{400}{150} = \frac{8}{3}$ **Ans.**

23. We are given a rectangle ABCD, where AB = 6 and BC = 3. Points F and G are on CD with DF = 1 and GC = 2. Lines AF and BG intersect at E and we are asked to find the area of triangle AEB.



Triangle AEB and triangle EFG are similar. DC = AB = 6 so FG = 6 – 3 = 3
Drop a perpendicular from point E to FG. This is the height of triangle EFG so let's call its length *h*. If we extend that

perpendicular all the way down to AB we have the height of triangle AEB and its length is $h + 3$.

$\frac{3}{6} = \frac{h}{h+3}$

$3 \times (h + 3) = 6h$

$3h + 9 = 6h$

$3h = 9$

$h = 3$

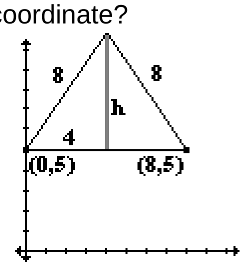
Therefore, the height of triangle AEB = $h + 3 = 6$.

The area of triangle AEB =

$\frac{1}{2} \times 6 \times 6 = 18$ **Ans.**

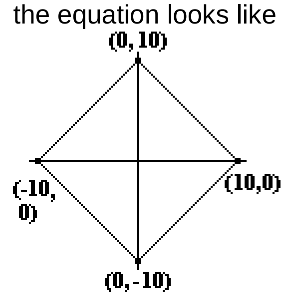
24. In a survey of 100 readers, three times as many people liked fiction as those who liked non-fiction. Eight liked both so how many people liked just non-fiction?
If we let x = those who liked non-fiction, then $3x$ = those who liked fiction.
Since the 8 are counted twice, we know that:
$x + 3x - 8 = 100$
$4x = 108$
$x = 27$ **Ans.**

25. An equilateral triangle has two vertices at (0, 5) and (8, 5). The third vertex is in the first quadrant so what is its y-coordinate?



Let A = (0, 5) and B = (8, 5), and let C be the third vertex. For triangle ABC to be equilateral, AB, BC, and AC must all be equal. BC and AC will be equal if and only if point C is equidistant from A and B. Thus, C must lie on the line y = 4. Dropping a perpendicular line from the top vertex to the base of the triangle gives us the height, h, and creates a right triangle whose hypotenuse is 8 and one side (opposite the 30º angle is $\frac{1}{2} \times 8 = 4$. We can now find the height.
$4^2 + h^2 = 8^2$
$16 + h^2 = 64$
$h^2 = 64 - 16 = 48$
$h = \sqrt{48} = 4\sqrt{3}$
However, to find the y-coordinate, remember that the base of the triangle is at $y = 5$ so we must add 5.
$5 + 4\sqrt{3}$ or $4\sqrt{3} + 5$ **Ans.**

26. We are asked to find the area enclosed by the graph of the equation
$|x| + |y| = 10$
If we let $y = 0$, we get the values 10 and –10 for x. If we let $x = 0$, we get the values 10 and –10 for y. The graph of the equation looks like this:



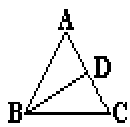
This is a square whose diagonals are 10 + 10 = 20
Let s = a side of the square. Then
$s^2 = 10^2 + 10^2 = 100 + 100 = 200$
Since s^2 is also the area of the square, the area is 200. **Ans.**

27. $f(x) = 5x - 3$
$g(x) = 3x^2 + 1$
$h(x) = f(x) + g(x)$
We are asked to find the sum of the x-values for which $h(x) = 0$.
$h(x) = f(x) + g(x) = 0$
$5x - 3 + 3x^2 + 1 = 0$
$3x^2 + 5x - 2 = 0$
$(3x - 1)(x + 2) = 0$
$x = \frac{1}{3}, -2$
$\frac{1}{3} + (-2) = \frac{1}{3} - \frac{6}{3} = -\frac{5}{3}$ **Ans.**

28. Two real numbers have an average of 7. Let x and y be the two real numbers.
Then $\frac{x+y}{2} = 7$ and $x + y = 14$.
The average of their squares is 54.
Thus, $\frac{x^2+y^2}{2} = 54$ and $x^2 + y^2 = 108$.
We are asked to find the value of xy.
$(x+y)^2 = x^2 + 2xy + y^2 = 14^2 = 196$
Notice, there is an x^2 and a y^2, so we can plug 108 in for $x^2 + y^2$.
$108 + 2xy = 196$
$2xy = 88$

$xy = 44$ **Ans.**

29. In triangle ABC, AB = AC. D is a point on AC so that BD bisects angle ABC. BD = BC and we are asked to find the measure of angle A.



Since AB = AC, triangle ABC is an isosceles triangle so the measure of angle ABC = the measure of angle BCA. Let x = angle BCA. We know that angle DBC is $\frac{x}{2}$ since BD bisects angle ABC.

That makes angle BDC =

$180 - \left(x + \frac{x}{2}\right) = 180 - \frac{3x}{2}$

Since BD = BC, triangle BCD is also isosceles with angle BCD = angle BDC.

$180 - \frac{3x}{2} = x$

$360 - 3x = 2x$

$360 = 5x$

$x = 72$

We are asked to find the measure of angle A = 180 – 2x = 180 – (2 × 72) = 180 – 144 = 36 **Ans.**

30. m and n are positive integers such that for some positive integer x, $\sqrt{x+43} = m$ and $\sqrt{x+16} = n$.

So what is the value of mn?

m and n are positive integers means that x + 43 is a square and x + 16 is a square. The difference between x + 16 and x + 43 is 27. If you write down the sequence of squares, you can see that the difference between two adjacent entries in the sequence is 2 larger than the previous 2, e.g., the difference between the 5^{th} and 6^{th} squares in the sequence is 2 more than the difference between the 4^{th} and 5^{th} squares in the sequence. All we have to do is list the squares in the sequence.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100
121, 144, 169, 196

196 – 169 = 27. So the smaller square is 13 and the larger is 14.

13 × 14 = 182 **Ans.**

(Note, that we actually don’t have to list out the entire sequence to find this. The difference between squares x and x + 1 in the sequence is 3 + 2(x – 1).

$3 + 2(x - 1) = 27$

$2x - 2 = 24$

$2x = 26$

$x = 13$…

So we can solve it either way.)

TARGET ROUND

1. Three golf balls and one bottle of water cost $12.00. One golf ball and four bottles of water cost $9.50. We are asked to find the cost of a bottle of water.

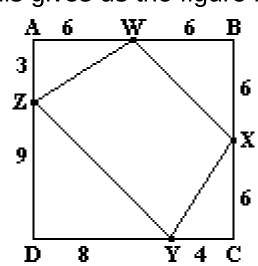
   Let g = the cost of a golf ball.
   Let w = the cost of a bottle of water.
   $3g + w = 12.00$
   $g + 4w = 9.50$
   Notice we have two variables and two equations. That means we can do substitution to solve!
   $g = 9.50 - 4w$
   $3(9.50 - 4w) + w = 12.00$
   $28.50 - 12w + w = 12.00$
   $28.50 - 11w = 12.00$
   $11w = 16.50$
   $w = 1.50$ **Ans.**

2. We are given a square ABCD with sides of length 12. Points W, X, Y and Z lie on sides AB, BC, CD and DA, respectively, so that

   AW = $\frac{1}{2}$ AB, BX = $\frac{1}{2}$ BC

   CY = $\frac{1}{3}$ CD, AZ = $\frac{1}{4}$ DA

   This gives us the figure below



   where AW = WB = 6,
   BX = XC = 6,
   DY = 8, YC = 4,
   AZ = 3, and ZD = 9.
   We are asked to find the area of quadrilateral WXYZ. We can do this by finding the area of the square and subtracting away the triangles that are formed at the corners.
   The area of square ABCD is
   12 × 12 = 144.
   Now for the area of the triangles…

Area of AWZ = $\frac{1}{2} \times 3 \times 6 = 9$
Area of ZDY = $\frac{1}{2} \times 9 \times 8 = 36$
Area of WBX = $\frac{1}{2} \times 6 \times 6 = 18$
Area of XYC = $\frac{1}{2} \times 6 \times 4 = 12$
9 + 36 + 18 + 12 = 75
144 – 75 = 69 **Ans.**

3. We are given a stem and leaf plot of 37 test scores as follows:
5| 8 9
6| 0 2 2 5 5 8 8
7| 0 1 2 2 3 5 5 5 6 8
8| 1 3 5 5 5 6 8 8 9 9
9| 0 1 2 3 8 8 9 9
We are asked to find the percent of the scores that are at most 5 points from the median?
First, let's find the median. Since there are 37 test scores, the median is the 19th score (i.e., the middle one since the number of scores is odd).
So the median is 78. 5 points below the median is 73. 5 points above the median is 83. The number of scores in the range of 73 to 83 is 8 (73, 75, 75, 75, 76, 78, 81, 83).
$\frac{8}{37}(100) \approx 21.621 \approx 22$ **Ans.**

4. $a + b + c + d = 11$
$2a + 3c = 19$
$b + 4d = 22$
$4a + d = 14$
$5b + 3c = 5$
So what is d? Let's start by adding together the last 4 equations.
$$\begin{array}{l} 2a + 3c = 19 \\ + b + 4d = 22 \\ +4a + d = 14 \\ \underline{+ 5b + 3c = 5} \\ 6a + 6b + 6c + 5d = 60 \end{array}$$
Notice that all of the variables have a coefficient of 6 except d. That means we can multiply the first equation by 6 and subtract from it the sum of the other 4 equations and will be left with d.
$(a + b + c + d = 11)(6) \rightarrow 6a + 6b + 6c + 6d = 66$
$$\begin{array}{r} 6a + 6b + 6c + 6d = 66 \\ \underline{-\ 6a + 6b + 6c + 5d = 60} \\ d = 6 \end{array}$$
Ans.

5. We are given an infinite sequence consisting of all positive integers, in increasing order that are neither multiples of 5 or 7. We are asked to find the 30th term.
From 1 to 10, we remove 5, 7 and 10 for a total of 7 terms that are part of the sequence.
From 11 to 20, we remove 14, 15, and 20 for a total of 7.
That's a total of 7 + 7 = 14 terms so far.
From 21 to 30, we remove 21, 25, 28, and 30 for a total of 6.
That's 20 terms so far.
From 31 to 40 we remove 35 and 40 for a total of 8.
That's 28 terms so far.
41 isn't a multiple of 5 or 7 so that's our 29th number.
42 is a multiple of 7.
43 isn't a multiple of 5 or 7 so that's the 30th element in the sequence.
43 **Ans.**

6. Brand A cornflakes are 50% more expensive than Brand B.
Brand A cornflakes weigh 25% more than Brand B.
We are asked to find, for equal weights, how much more Brand A costs than Brand B, as a percentage. Let's pick numbers.
If we let the price of Brand B be \$4, then the price of Brand A is \$6. If we let the weight of Brand B be 4 pounds, then the weight of the box of Brand A is 5 pounds.
Now we can look at the unit price of each. Brand B costs \$4/4 lbs = \$1 per lb. Brand A costs \$6/5 lbs = \$1.20 per lb. That is a difference of \$0.20 per lb, which is (0.2/1)(100) = 20% **Ans.**

7. Two coins are randomly placed in different squares in a 4-by-4 grid. We are asked to find the probability that the two coins do not lie in the same row or column.
There are 16 squares in a 4-by-4 grid. When a coin is placed in the grid there are a total of 15 that remain unoccupied, 3 of which are in the same row and 3 of which are in the same column. That's means

that of the 15 remaining squares 3 + 3 = 6 are out of bounds. That leaves 15 – 6 = 9 squares that are okay. Thus, the probability that the second coin is not in the same row or column as the first coin is

$\frac{9}{15} = \frac{3}{5}$ **Ans.**

8. How many ordered pairs of positive integers satisfy the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$, with $x < y$?

   Let's first get the value of y in terms of x.

   $\frac{1}{y} = \frac{1}{6} - \frac{1}{x} = \frac{x-6}{6x}$

   $y = \frac{6x}{x-6}$

   Since x and y must be positive, x must be greater than 6.

   If x is 7, $\frac{42}{1}$ = 42, and (7, 42) is an ordered pair.

   If x is 8, $\frac{48}{2}$ = 24, and (8, 24) is an ordered pair.

   If x is 9, $\frac{54}{3}$ = 18, and (9, 18) is an ordered pair.

   If x Is 10, $\frac{60}{4}$ = 15, and (10, 15) is an ordered pair.

   If x is 11, $\frac{66}{5}$ does not resolve to an integer so 11 is not part of an ordered pair.

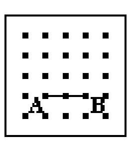
   If x is 12, $\frac{72}{6}$ = 12 and (12, 12) is an ordered pair. NO! = What a minute! It can't be because x must be less than y. If x is any larger, y will now be smaller so we are done.

   Thus, there are 4 ordered pairs.

   4 **Ans.**

TEAM ROUND

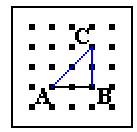
1. Segment AB is drawn on the geoboard as shown.

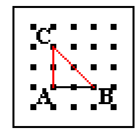


   Point C is to be chosen from the remaining 23 points. We are asked to find the number of points for C that will cause an isosceles triangle to be drawn.

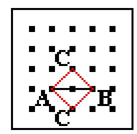
   Let's start with AB being one of the two segments of triangle ABC that are the same length. The length of AB is 2.

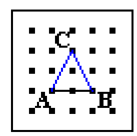
   The first figure shows that we can make AB and BC the same. The second figure shows that we can make AB and AC the same. That's two points.

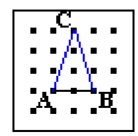




   The next three figures show that we can use 4 points for C that make CA and CB the same.







   2 + 4 = 6 **Ans.**

2. A sequence of letters consists entirely of X's and Y's where X's always appear consecutively in groups of an even number of X's while Y's always appear consecutively in groups of a multiple of three Y's. We are asked to find how many different sequences of 13 letters are possible.

   Using combinations of 2's and 3's, how many different ways can we get the sum of 2's and 3's to result in 13?

   2 + 2 + 2 + 2 + 2 + 3 = 13

   2 + 2 + 3 + 3 + 3 = 13

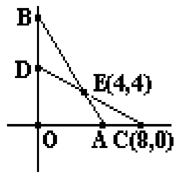
   The first combination can appear in 6 different ways.

   The second combination can appear in $\frac{5!}{2!3!} = \frac{5\times4}{2\times1} = 10$ different ways.

   10 + 6 = 16 **Ans.**

3. A line with a slope of –2 intersects the positive x-axis at A and the positive y-axis at B. A second line

interests the x-axis at C(8, 0) and y=the y-axis at D. The two lines intersect each other at E(4, 4). So what is the area of the quadrilateral OBEC?



The first line, AB, has a slope of –2 and one of the points on it is E(4, 4).

$y = mx + b$

$m = –2$

$4 = (–2 \times 4) + b$

$4 = –8 + b$

$b = 12$

Therefore, the equation of the first line is

$y = –2x + 12$

Let's find the coordinates of point A.

$0 = –2x + 12$

$2x = 12$

$x = 6$

The coordinates of A are (6, 0).

Now let's find the coordinates of point B.

$y = 0 + 12$

$y = 12$

The coordinates of B are (0, 12).

Finally, let's determine the equation for line CD. We have the points C and E to use for this.

$y = mx + b$

$0 = 8m + b$

$4 = 4m + b$

$–4 = 4m$

$m = –1$

$0 = –8 + b$

$b = 8$

So the equation for this line is

$y = –x + 8$

Now we can get the coordinates for point D.

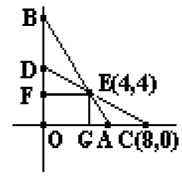
$y = mx + b$

$y = 0 + 8$

$y = 8$

The coordinates of point D are (0, 8).

Now lets add point F at (0, 4) and G at (4, 0).



The area of square FEGO is 4 × 4 = 16.

BF = BD + DF = 4 + 4 = 8

The area of triangle BFE is

$\frac{1}{2} \times 8 \times 4 = 16$

GC = GA + AC = 2 + 2 = 4

The area of triangle EGC is

$\frac{1}{2} \times 4 \times 4 = 8$

16 + 16 + 8 = 40 **Ans.**

4. What is the sum of the cubes of the first 10 positive odd integers?
   Okay. That's nasty…But it's a calculator round. Just be careful entering in the values!
   $1^3 + 3^3 + 5^3 + 7^3 + 9^3 + 11^3 + 13^3 + 15^3 + 17^3 + 19^3 =$
   1 + 27 + 125 + 343 + 729 +
   1331 + 2197 + 3375 + 4913 + 6859
   = 1225 + 18,675 = 19,900 **Ans.**

5. A set of five positive integers has a mean, median and range of 7. How many distinct sets could have these properties?
   The third number must be 7 since the median of an odd number of integers is just the middle integer. The difference between the first and fifth number is 7. And the sum of all 5 numbers must be 35.
   Since the numbers must be positive, the smallest could be 1 and the largest 8. Can we create a set that qualifies?
   1 + 7 + 8 = 16
   That would leave 35 – 16 = 19 for the sum of the second and fourth integers. No, this won't work.
   What about the smallest at 2 and the largest at 9.
   2 + 7 + 9 = 18
   That leaves 35 – 18 = 17 for the sum of the second and fourth integers. This won't work either.
   What about the smallest at 3 and the largest at 10?
   3 + 7 + 10 = 20
   That leaves 35 – 20 = 15 for the

sum of the second and fourth numbers. This can work a few ways:
{3, 5, 7, 10, 10}
{3, 6, 7, 9, 10}
{3, 7, 7, 8, 10}
What about the smallest at 4 and the largest at 11?
4 + 7 + 11 = 22
That leaves 35 – 22 = 13 for the sum of the second and fourth numbers. This can work a few ways:
{4, 4, 7, 9, 11}
{4, 5, 7, 8, 11}
{4, 6, 7, 7, 11}
What about the smallest at 5 and the largest at 12?
5 + 7 + 12 = 24
That leaves 35 – 24 = 11 for the sum of the second and fourth numbers. This is now too small.
That gives us 6 sets. **Ans.** 6

6. $r \# s = r^2 - s^2$
$6 \# 4 = 6^2 - 4^2 =$
$36 - 16 = 20$ **Ans.**

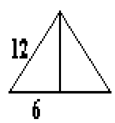
7. What is the greatest common factor of 20! and 200,000?
Let's first look at the prime factorization of 200,000.
$200{,}000 = 2 \times 10^5 =$
$2 \times 2^5 \times 5^5 = 2^6 \times 5^5$
Notice that 20! has 10 even numbers so all 6 2's from 200,000 can also be found in 20!.
How many 5's does 20! have?
One each for 5, 10, 15, and 20.
That's a total of 4 5s. That means there is one extra 5 in 200,000 that must be removed.
$\frac{200{,}000}{5} = 40{,}000$ **Ans.**

8. Harry has 49 coins totaling $1.00. The coins consist of pennies, nickels, dimes and/or quarters. We are asked to find the smallest number of dimes he could have.
Let's start by trying 0 dimes.
Suppose Harry has 3 quarters. That leaves 25 cents to go. But 3 + 25 (if you're using pennies) = 28 coins and this won't work.
Suppose that Harry had 2 quarters. He could use 50 pennies but that means 50 + 2 = 52 coins. What about using 1 dime?
That would leave 40 pennies and 2 + 1 + 40 = 43 coins. No, that won't do either.
Suppose that Harry has only 1 quarter. We know he can't have 75 pennies and 0 dimes. So let's try one dime. So far so good. We need 47 coins to make up $.65 to get him to $1.00. And they must be nickels and pennies.
1 nickel + 60 pennies. No.
2 nickels + 55 pennies. No.
3 nickels + 50 pennies. No.
4 nickels + 45 pennies. No.
5 nickels + 40 pennies. No, that's too few coins.
So try 2 dimes.
That's 3 coins and we need to 46 more coins to make up $0.55 cents.
1 nickel + 50 pennies. No.
2 nickels + 45 pennies. No.
3 nickels + 40 pennies. Too little.
So let's try 3 dimes. That means we need 45 coins to make up $0.45.
0 nickels + 45 pennies. That will do it. So 3 is a possibility.
So what about if we don't have any quarters at all? Can we get a better result? No. We can actually get to $1.00 by using 40 pennies, 6 nickels and 3 dimes, but this case also requires 3 dimes.
3 **Ans.**

9. A bag has 3 red balls, 4 green balls and 5 yellow balls. If balls are drawn one at a time without replacement, what is the probability that the first yellow ball is drawn on the eighth draw?
We have 3 + 4 = 7 non-yellow balls, so they must all be drawn first.
There are a total of 3 + 4 + 5 = 12 balls.
$\frac{7\times6\times5\times4\times3\times2\times1}{12\times11\times10\times9\times8\times7\times6} =$
$\frac{5\times4\times3\times2}{12\times11\times10\times9\times8} =$
$\frac{1}{11\times9\times8} = \frac{1}{792}$ **Ans.**

10. The sum of the perimeters of two equilateral triangles is 45 inches. The area of the larger one is 16 times the area of the smaller one.

So what is the area of the larger triangle?
Let x = the side of the first triangle.
Let y = the side of the second triangle.
$3x + 3y = 45$
$x + y = 15$
$y^2 = 16x^2$
$y = 4x$
$x + 4x = 15$
$x = 3$
$4x = 12$
So the side of the larger triangle is 12. That gives us a triangle that looks like this:



Let h be the altitude.
$h^2 + 6^2 = 12^2$
$h^2 + 36 = 144$
$h^2 = 144 – 36 = 108$
h = $\sqrt{108} = 2\sqrt{27} = 6\sqrt{3}$
(This can be done more quickly if you remember that the side opposite the 60º angle is $\frac{s\sqrt{3}}{2}$)
The area is:
$\frac{1}{2} \times 12 \times 6\sqrt{3} = 36\sqrt{3}$ **Ans.**