MATHCOUNTS®

2011
■ Chapter Competition ■
Countdown Round
Problems 1–80

This section contains problems to be used in the Countdown Round.

National Sponsors

Raytheon Company * National Defense Education Program *
Northrop Grumman Foundation *
National Society of Professional Engineers * CNA Foundation *
ThinkFun * Texas Instruments Incorporated * 3M Foundation

Raytheon

2011 MATHCOUNTS
National Competition Sponsor

Founding Sponsors: National Society of Professional Engineers, National Council of Teachers of Mathematics and CNA Foundation

1. Meera began an exam at 11:37 a.m. and finished at 1:19 p.m. the same day. How many minutes did she take to complete the exam?

1. _____ (minutes)

2. If $3x + 8 = 23$, what is the value of $3x - 3$?

2. _____

3. Three out of every five students at Pythagoras Middle School went to the Spring Fling. If 202 students did not go to the Spring Fling, how many students attend this school?

3. _____ (students)

4. Each edge length of a cube is tripled. How many times the volume of the original cube is the volume of the new cube?

4. _____ (times)

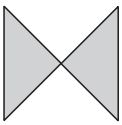
5. What is the sum of the three prime numbers between 40 and 50?

5. _____

6. If x is an integer such that $9 > x > 2$, what is the greatest value of $5x + 11$?

6. _____

7. Two congruent isosceles right triangles are joined to create this figure. Each leg of each triangle measures 15 units. What is the total area of the figure, in square units?



7. _____ (sq units)

8. If the sum of the digits of a positive two-digit integer is 15, what is the probability that one of the digits is 9? Express your answer as a common fraction.

8. _____

9. A right triangle has a hypotenuse of 26 units. If one leg is 4 more than twice the other, what is the sum of the lengths of the legs, in units?

9. _____ (units)

10. The length of a right, rectangular prism is doubled, its width is quadrupled and its height is unchanged. What is the ratio of the original volume to the new volume? Express your answer as a common fraction.

10. _____

11. If $n + (1/n) = 5$, what is the value of $n^2 + (1/n^2)$?

11. _____

12. The endpoints of segment BC are B(7, 4) and C(3, 6). What is the product of the coordinates of the midpoint of segment BC?

12. _____

13. If $m \blacktriangle n = 2m + n$ and $m \blacktriangledown n = 4m - n$, what is the value of $(2 \blacktriangle (5 \blacktriangledown 3))$?

13. _____

14. Given $a + b = 23$, $b + c = 35$ and $a + c = 20$, what is the average of a, b and c?

14. _____

15. What is the sum of the distinct prime divisors of $11 + 11^2$?

15. _____

16. The product of three consecutive integers is 990. What is their sum? 16. _____

17. If $3^n = 9^3$, what is the integer value of n? 17. _____

18. If a 12-hour analog clock reads 8:00, what is the degree measure of the smaller angle formed by the minute and hour hands? 18. _____ (degrees)

19. The probability that Joshua will draw a blue marble at random from a bag containing yellow marbles and blue marbles is $\frac{2}{3}$. If 14 of the marbles are yellow, how many marbles are in the bag before any marbles are removed? 19. _____ (marbles)

20. What is the area, in square units, of a triangle whose vertices are at (4, –1), (10, 3) and (4, 5)? 20. _____ (sq units)

21. What is the value of the digit K that will make the number 481,5K6 divisible by 2, 3, 4 and 9? 21. _____

22. Jamie can paint a wall in 20 minutes, and Ellie can paint the same size wall in 15 minutes. If they work together, how many minutes will it take them to paint one wall of this size? Express your answer to the nearest whole minute. 22. _____ (minutes)

23. When four consecutive integers are added, the result is –18. What is the greatest product that can be obtained by multiplying two of these four integers? 23. _____

24. The GCF of two numbers is 70. Their product is 49,000. What is the LCM of the two numbers? 24. _____

25. Ten distinct points are arranged on a circle. How many different triangles are there whose three vertices are among those ten points? 25. _____ (triangles)

26. A store advertised a computer at 50% off plus an additional 20% off the sale price. On top of that, Erich had a coupon for 10% off the final price. If he paid $360 before taxes, what was the original price of the computer? 26. _____ (dollars)

27. Mellie bought a box of dog biscuits for her golden retriever Karma. She gives Karma 3 dog biscuits a day. After ten days she counted 96 biscuits left in the box. At this rate, for how many weeks will a full box of dog biscuits feed Karma? 27. _____ (weeks)

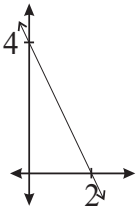
28. Miss Minerva challenges her class to guess her secret positive integer. She gives them these clues: The sum of the digits of this three-digit number is 14. The tens digit is the cube of a prime number. The units digit is one-fifth of the hundreds digit. What is Miss Minerva's number? 28. _____

29. What is the maximum number of 2-inch by 2-inch by 2-inch cubes that can be placed in a box that measures 7-inches by 8-inches by 9-inches? 29. _____ (cubes)

30. How many different games need to be played so that 6 teams each play each other exactly once? 30. _____ (games)

31. Troy started with a solid 3 by 3 by 3 cube. To each face of this 3 by 3 by 3 cube, he attached one unit cube by pasting one face of the unit cube to the face of the larger cube. What is the percent of increase in the total surface area of this structure? Express your answer to the nearest whole percent. 31. _____ (percent)

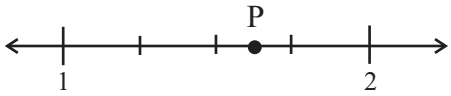
32. The graph of the line $y = mx + b$ is shown. What is the value of $b - m$? 32. _____



33. If $4x + y = -12$ and $x + 4y = -3$, what is the value of $3x + 3y$? 33. _____

34. What is the arithmetic mean of the first five positive integers that are powers of 10? Express your answer as a mixed number. 34. _____

35. The number line shown has uniformly-spaced markings. If point P is equidistant from its two closest markings, what is the coordinate of P? Express your answer as a common fraction. 35. _____



36. What is the smallest positive integer that can be added to the sum of the consecutive integers $(1 + 2 + \ldots + 10 + 11)$ so that the resulting total is divisible by 5? 36. _____

37. If the product $(2^2)(3^3)(4^4)(5^5)(6^6)(7^7)(8^8)(9^9)(10^{10})$ is written as an integer, how many zeros are to the right of the right-most non-zero digit? 37. _____ (zeros)

38. When the decimal point is moved one place to the left in a two-digit positive integer, the difference between these two numbers is 72.9. What is the sum of the two numbers? Express your answer as a decimal to the nearest tenth. 38. _____

39. How many positive two-digit integers are 27 more than the sum of their digits?

39. _____ (integers)

40. When 60 is increased by 60% it then is equal to what number decreased by 60%?

40. _____

41. Two fair, standard six-sided dice are rolled. What is the probability that one of the two numbers is a factor of the other number? Express your answer as a common fraction.

41. _____

42. Given a right triangle whose side lengths are all integer multiples of 8, how many units are in the smallest possible perimeter of such a triangle?

42. _____ (units)

43. The average age of six people in a room is 30 years. A 20-year-old person leaves the room. What is the average age of the five remaining people, in years?

43. _____ (years)

44. If the ratio of $3x$ to y is $\frac{5}{6}$, what is the ratio of x to $3y$? Express your answer as a common fraction.

44. _____

45. How many integers x, such that $1 \leq x \leq 1000$, are multiples of 7?

45. _____ (integers)

46. What is the greatest common factor of 400 and 250?

46. _____

47. Right triangle ABC has vertices at A(1, 3), B(5, 3) and C(5, 1). If this triangle is reflected over the x-axis and the resulting triangle is then reflected over the y-axis, what are the coordinates of the final image of point A? Express your answer as an ordered pair

47. _____ (,)

48. The width of a rectangle is equal to the side length of a square. If the ratio of the area of the square to the area of the rectangle is 1:3, and the length of the rectangle is 15 units, what is the side length of the square, in units?

48. _____ (units)

49. What is the product of $\sqrt[4]{25}$ and $\sqrt[6]{125}$?

49. _____

50. What is the maximum number of non-overlapping interior regions that can be created by four chords of a circle?

50. _____ (regions)

51. Everyone at the party shook hands with everyone else exactly once. If there were a total of 21 handshakes, how many people were at the party?

51. _____ (people)

52. What fraction of 2 days is 30 minutes? Express your answer as a common fraction. 52. _____

53. An isosceles triangle has side lengths 8 cm, 8 cm and 10 cm. The longest side of a similar triangle is 25 cm. What is the perimeter of the larger triangle, in centimeters? 53. _____ (cm)

54. The integer A67,83B where A and B are the first and last digits of the integer, respectively, is divisible by 15. What is the largest possible sum of A and B? 54. _____

55. What is the degree measure of the supplement of the complement of a 42-degree angle? 55. _____ (degrees)

56. The prime factorization of N is $3^3 \times 7^R$. If N has exactly 16 positive integer factors, what is the value of R? 56. _____

57. What is the value of the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)...\left(1+\frac{1}{2011}\right)$? 57. _____

58. What is the sum of the two products 199×7 and 199×3? 58. _____

59. If $\frac{2}{5}$ of Y is 36, what is $\frac{5}{3}$ of Y? 59. _____

60. A square and a right triangle have equal perimeters. The legs of the right triangle are 20 inches and 15 inches. What is the area of the square, in square inches? 60. _____ (sq inches)

61. If $3^{3x} = 81$, then what is the value of x? Express your answer as a common fraction. 61. _____

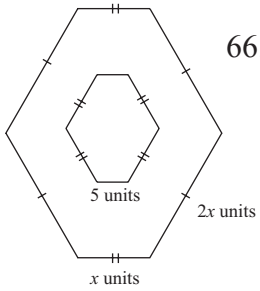
62. If the numerator of a positive fraction is multiplied by 8 and the denominator is divided by 4, how many times larger does the fraction become? 62. _____ (times)

63. If the legs of a right triangle measure 0.3 units and 0.4 units in length, what is the length of the hypotenuse, in units? Express your answer as a decimal to the nearest tenth. 63. _____ (units)

64. In a marching band, there are 68 woodwinds, 85 brass and 34 percussion players. When marching, the band must have the same number of players in each row, and each row must contain only players of one type of instrument. What is the fewest number of rows that can be formed when marching? 64. _____ (rows)

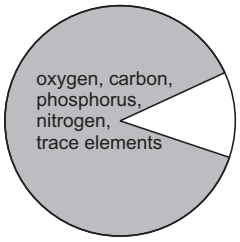
65. The product of two consecutive integers is less than or equal to 448. What is the largest possible sum of two such integers? 65. _____

66. The two hexagons shown are similar. Each segment marked with a double tick mark is x units long and each segment marked with a single tick mark is $2x$ units long. The two unmarked segments are each 5 units long. What is the positive difference of the perimeters of the two hexagons, in units? 66. _____ (units)



67. Circle O is located on the coordinate plane with center at (2, 3). One endpoint of a diameter is at (–1,–1). What are the coordinates of the other endpoint of this diameter? Express your answer as an ordered pair. 67. (,)

68. The human body is composed of 65% oxygen, 18% carbon, 1% phosphorus, 3% nitrogen and 1% trace elements. These are all shown in the gray region. Hydrogen and calcium make up the remainder of the pie graph. If there is five times as much hydrogen as calcium, what percent of the human body is hydrogen? 68. _____ (percent)



69. What is the largest product of two primes that have a sum of 60? 69. _____

70. The average of four consecutive odd integers is 26. What is the sum of the least and greatest of these integers? 70. _____

71. How many positive integer divisors does 48 have? 71. _____ (divisors)

72. One Saturday Gary noticed that his baby sister was 111 days old. On what day of the week was his baby sister born? 72. _____

73. When a positive integer is divided by 7, the result is 35 with a remainder of 5. What is the integer? 73. _____

74. The area of square ABCD is 8 square units. What is the area, in square units, of circle O, which is inscribed in the square? Express your answer in terms of π. 74. _____ (sq units)

75. How many positive factors does 30 have? 75. _____ (factors)

76. If 15 is $\frac{3}{5}$ of N, what is the value of N? 76. _____

77. The integer 144 has 15 factors. How many factors of 144 are greater than $\sqrt{144}$? 77. _____ (factors)

78. What is the sum of 10 consecutive positive integers that have a mean of k? Express your answer in terms of k.

78. _____

79. At Washington Middle School there are 900 students. If $\frac{3}{5}$ of the students are boys, what is the ratio of girls to boys? Express your answer as a common fraction.

79. _____

80. The measure of an exterior angle of a regular polygon is 90 degrees. How many sides does the polygon have?

80. _____ (sides)