

---

---

# MATHCOUNTS<sup>®</sup>

---

2011

■ State Competition ■  
Countdown Round  
Problems 1–80

---

**This section contains problems to be used in  
the Countdown Round.**

## **National Sponsors**

---

Raytheon Company \* National Defense Education Program \*  
Northrop Grumman Foundation \*  
National Society of Professional Engineers \* CNA Foundation \*  
ThinkFun \* Texas Instruments Incorporated \* 3M Foundation

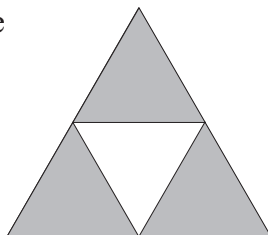
**Raytheon**

2011 MATHCOUNTS  
National Competition Sponsor

Founding Sponsors: National Society of Professional Engineers, National Council of Teachers of Mathematics and CNA Foundation

Copyright MATHCOUNTS, Inc. 2011. All rights reserved.

1. A jackpot for a contest starts at \$100,000 and increases 10% after each round. How many dollars is the jackpot worth after four rounds? 1. \_\_\_\_\_ (dollars)
2. What is the sum of the cubes of the first three even positive integers? 2. \_\_\_\_\_
3. What is the area, in square units, of a square whose sides are the same length as the radius of a circle with a circumference of  $12\pi$  units? 3. \_\_\_\_\_ (sq units)
4. What is the area, in square units, of the triangle bounded by  $y = 0$ ,  $y = x + 4$  and  $x + 3y = 12$ ? 4. \_\_\_\_\_ (sq units)
5. The population of the town of Leibnitz was 180,000 in 2002. If the population grows at a rate of 3000 people per year, what will the population be in 2020? 5. \_\_\_\_\_ (people)
6. If  $x$  and  $y$  are positive integers such that  $4x + 2y = 36$  and  $3y - 2x = 14$ , what is the value of the product  $xy$ ? 6. \_\_\_\_\_
7. How many integers are in the solution of the inequality  $|x + 4| < 9$ ? 7. \_\_\_\_\_ (integers)
8. A set of nine distinct positive integers has mean 9 and median 9. What is the greatest possible value of one of these integers? 8. \_\_\_\_\_
9. For how many positive integers  $N$  does  $\sqrt{N}$  differ from 10 by less than 2? 9. \_\_\_\_\_ (integers)
10. What is the value of the expression  $40(3000) - 50(400) + 18(20) - 20(5)$ ? 10. \_\_\_\_\_
11. Jack takes twice as long to dig a hole as Ken. The two working together can dig a hole in ten minutes. How many minutes does it take Ken to dig a hole by himself? 11. \_\_\_\_\_ (minutes)
12. Point  $P(-5, 12)$  is graphed in a coordinate plane. What is the number of units in the distance from point  $P$  to the origin? 12. \_\_\_\_\_ (units)
13. A region consists of an equilateral triangle divided into smaller congruent equilateral triangles. What percent of the region is gray? 13. \_\_\_\_\_ (percent)



14. What is the measure, in units, of the hypotenuse of a right triangle with leg lengths of 75 and 100 units?

14. \_\_\_\_\_ (units)

15. Mary will randomly choose an integer from the integers 1 to 100, inclusive. If Mary chooses a multiple of 4, what is the probability she will choose a perfect square? Express your answer as a common fraction.

15. \_\_\_\_\_

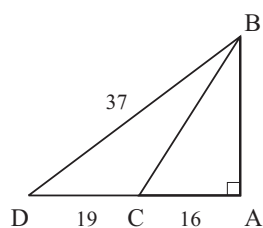
16. What is the value of  $x$  if  $6^x + 6^x + 6^x + 6^x + 6^x + 6^x = 6^6$ ?

16. \_\_\_\_\_

17. Amy drove from Mathtown to Mathville. Her average speed for the first two hours was 35 mph due to road construction. She then averaged 70 mph for the rest of the trip. If her average speed for the entire trip was 60 mph, how many hours did the trip take?

17. \_\_\_\_\_ (hours)

18. Given the right triangles ABC and ABD, what is the length of segment BC, in units?

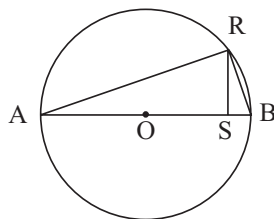


18. \_\_\_\_\_ (units)

19. Given 3 consecutive positive integers, what is the positive difference between the product of the first and last integers and the square of the middle integer?

19. \_\_\_\_\_

20. Circle O has diameter AB of length 26 units. Segment RS is perpendicular to segment AB with R on the circle and S on segment AB. If the length of segment RS is 12 units, what is the product of the lengths of segments AS and SB, in square units?

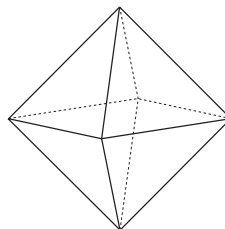


20. \_\_\_\_\_ (sq units)

21. What is the integer value of  $\left(\left(\sqrt{2}\right)^{\sqrt{2}}\right)^{\sqrt{8}}$ ?

21. \_\_\_\_\_

22. Two of the vertices of a regular octahedron are to be chosen at random. What is the probability that they will be the endpoints of an edge of the octahedron? Express your answer as a common fraction.

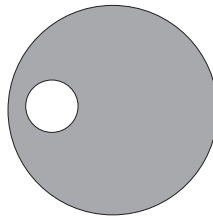


22. \_\_\_\_\_

23. Two successive discounts of 20% are equivalent to a single discount of what percent?

23. \_\_\_\_\_ (percent)

24. What is the area of the gray region, in square units, if the radius of the larger circle is four times the radius of the smaller circle and the diameter of the smaller circle is 2 units? Express your answer in terms of  $\pi$ .



24. \_\_\_\_\_ (sq units)

25. The sum of five consecutive integers is 185. What is the mean of these five integers?

25. \_\_\_\_\_

26. A regular octagon is graphed in the coordinate plane. Two adjacent vertices are graphed at  $(4, -2)$  and  $(1, 2)$ . What is the perimeter of this octagon, in units?

26. \_\_\_\_\_ (units)

27. Joey and Mike live 300 miles from each other at opposite ends of State Highway 1. Driving toward each other on State Highway 1, they met in the central part of the state. Joey left home at 9:30 a.m. traveling at an average speed of 50 mph and Mike left home at 10:00 a.m. traveling at an average speed of 60 mph. At what time that afternoon did they meet?

27. \_\_\_\_\_ : \_\_\_\_\_ (p.m.)

28. Four fair coins are to be flipped. What is the probability that all four will be heads or all four will be tails? Express your answer as a common fraction.

28. \_\_\_\_\_

29. The arithmetic mean of eight positive integers is 7. If one of the eight integers is removed, the mean becomes 6. What is the value of the integer that is removed?

29. \_\_\_\_\_

30. What is the smallest number of people you would need to have in a room if you wanted to be certain that at least 25 of them have the same birth month?

30. \_\_\_\_\_ (people)

31. On her previous five attempts Sarah had achieved times, in seconds, of 86, 94, 97, 88 and 96, for swimming 50 meters. After her sixth try she brought her median time down to 92 seconds. What was her time, in seconds, for her sixth attempt?

31. \_\_\_\_\_ (seconds)

32. What is the integer value of  $78^2 + 78 \times 22 + 22^2$ ?

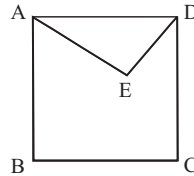
32. \_\_\_\_\_

33. Marty bought a tie that was on sale for 20% off, and he used a coupon that took off an additional 40% from the sale price. His son Jerry bought a tie that was on sale for 60% off with no coupon. If both ties originally cost \$50, how much more did Marty spend on a tie than Jerry, in dollars?

33. \_\_\_\_\_ (dollars)

34. What is one-half of the reciprocal of  $\frac{7}{\sqrt{98}}$ ? Express your answer in the form  $\frac{\sqrt{a}}{b}$  where  $\sqrt{a}$  is in simplest radical form. 34. \_\_\_\_\_

35. Quadrilateral ABCD is a square, and segment AE is perpendicular to segment ED. If AE = 8 units and DE = 6 units, what is the area of pentagon AEDCB, in square units? 35. \_\_\_\_\_ (sq units)



36. A  $2 \times 2 \times 2$  cube is removed from each corner of an  $8 \times 8 \times 8$  cube. What fraction of the original cube remains? Express your answer as a common fraction. 36. \_\_\_\_\_

37. What is the sum of the roots of  $x^2 - 4x + 3 = 0$ ? 37. \_\_\_\_\_

38. If the sum of  $0.\overline{4} + 0.0\overline{4} + 0.00\overline{4}$  is written as a fraction with a denominator of 300, what is the numerator? 38. \_\_\_\_\_

39. How many positive common fractions with a value less than  $\frac{1}{2}$  have 37 as their denominator? 39. \_\_\_\_\_ (fractions)

40. If  $\frac{4}{3}(r + s + t) = 12$ , what is the average of  $r$ ,  $s$  and  $t$ ? 40. \_\_\_\_\_

41. The measure of the supplement of angle A is six times the measure of the complement of angle A. What is the measure, in degrees, of angle A? 41. \_\_\_\_\_ (degrees)

42. In a survey of 100 students who watch television, 21 watch American Idol, 39 watch Lost, and eight watch both. How many of the students surveyed watch neither show? 42. \_\_\_\_\_ (students)

43. The area of an equilateral triangle is numerically equal to the length of one of its sides. What is the perimeter of the triangle, in units? Express your answer in simplest radical form. 43. \_\_\_\_\_ (units)

44. When the sum of the reciprocals of two distinct positive integers is divided by the sum of the two integers, the result is  $\frac{1}{25}$ . What is the sum of the two integers? 44. \_\_\_\_\_

45. If  $x$  varies inversely with  $y$  and  $y$  varies directly with the square of  $z$ , then by what positive factor is  $z$  multiplied when  $x$  is multiplied by one-fourth? 45. \_\_\_\_\_

46. To make orange juice from concentrate, the directions call for mixing one 12-ounce can of concentrate with five 12-ounce cans of water. A serving of orange juice is 8 ounces. If only whole cans can be used, what is the minimum number of cans of concentrate that Willy will need for at least 130 servings? 46. \_\_\_\_\_ (cans)

47. The side length of square A is 36 cm. The side length of square B is 42 cm. What is the ratio of the area of square A to the area of square B? Express your answer as a common fraction. 47. \_\_\_\_\_

48. If  $n$  is the smallest integer greater than the reciprocal of  $0.272727$ , what is  $n$ ? 48. \_\_\_\_\_

49. The real numbers  $x$ ,  $y$  and  $z$  satisfy the equations  $x + 2y + 3z = 950$  and  $3x + 2y + z = 1450$ . What is the average of  $x$ ,  $y$  and  $z$ ? 49. \_\_\_\_\_

50. How many integers between 211 and 2101 are multiples of 7? 50. \_\_\_\_\_ (integers)

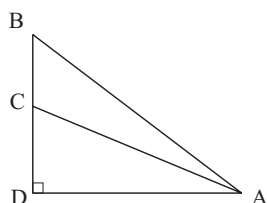
51. If two numbers will be randomly chosen without replacement from  $\{3, 4, 5, 6\}$ , what is the probability that their product will be a multiple of 9? Express your answer as a common fraction. 51. \_\_\_\_\_

52. For every  $3^\circ\text{C}$  rise in temperature, the volume of a certain gas increases by 4 cubic centimeters. If 50 cubic centimeters of this gas at  $-8^\circ\text{C}$  is heated to  $28^\circ\text{C}$ , by what percent does the volume increase? 52. \_\_\_\_\_ (percent)

53. The three-digit integer  $1AB$  equals  $1! + A! + B!$ . What is  $A + B$ ? 53. \_\_\_\_\_

54. A bag contains only blue marbles, green marbles and 24 red marbles. If the probability of drawing a blue marble is  $\frac{1}{2}$  and the probability of drawing a green marble is  $\frac{1}{8}$ , how many marbles are in the bag? 54. \_\_\_\_\_ (marbles)

55. What is the area, in square units, of triangle ABC in the figure shown if points A, B, C and D are coplanar, angle D is a right angle,  $AC = 13$ ,  $AB = 15$  and  $DC = 5$ ?



55. \_\_\_\_\_ (sq units)

56. The sum of three consecutive integers is greater than 66. What is the smallest possible product of the largest and smallest of these integers? 56. \_\_\_\_\_

57. If  $g(x) = 3x + 7$  and  $f(x) = 5x - 9$ , what is the value of  $f(g(8))$ ? 57. \_\_\_\_\_

58. How many diagonals does a convex polygon with 23 sides have? 58. \_\_\_\_\_ (diagonals)
59. What is the sum of all real solutions to the equation:  
 $\sqrt{x + \sqrt{9 - x}} = 3$ ? 59. \_\_\_\_\_
60. Mr. George has 25% more students this year than he had last year. He has 120 students this year. How many students did he have last year? 60. \_\_\_\_\_ (students)
61. If  $(x^2 - 5y)^3$  is written in expanded form, what is the coefficient of the  $x^2y^2$  term? 61. \_\_\_\_\_
62. What is the smallest prime that is the sum of four different, positive composite integers? 62. \_\_\_\_\_
63. Triangle ABC is an isosceles triangle with side lengths of 25, 25 and 48 centimeters. What is the area of triangle ABC, in square centimeters? 63. \_\_\_\_\_ (sq cm)
64. A line passes through A(8, 4) and B(-12, -11). What is the value of  $x$  if  $(x, -5)$  is on the line? 64. \_\_\_\_\_
65. Let # be the relation defined by  $A \# B = A^2 + B^2$ . If  $A \# 5 = 169$ , what is the positive value of A? 65. \_\_\_\_\_
66. Triangle ABC with vertices at A(1, 1), B(1, -2) and C(5, -2) is translated up 3 units and then dilated with respect to the origin by a factor of 2. What are the new coordinates of point C? Express your answer as an ordered pair. 66. \_\_\_\_\_ ( , )
67. Major Domo wishes to donate a sum of money to each of his three favorite charities. His total donation is to be divided among the charities in a ratio of 5:3:2. If his total gift is \$20,000, what is the difference, in dollars, between the largest donation and the smallest donation? 67. \_\_\_\_\_ (dollars)
68. If  $n$  is a positive integer, what is the smallest value of  $n$  for which  $\sqrt{\frac{8n}{5}}$  is an integer? 68. \_\_\_\_\_
69. Triangle ABC is drawn inside regular hexagon ABCDEF. What is the ratio of the area of triangle ABC to the area of the hexagon? Express your answer as a common fraction. 69. \_\_\_\_\_

70. Jamie has 2 dimes, 4 nickels and 8 pennies. In how many different ways can she make 26¢? 70. \_\_\_\_\_ (ways)
71. What is the product of all integer values of  $x$  for which the value of  $|x^2 - 9|$  is a prime number? 71. \_\_\_\_\_
72. If  $a$  is  $\frac{2}{3}$  of  $b$  and  $b$  is  $\frac{1}{4}$  of  $c$ , what fraction of  $c$  is  $a$ ? Express your answer as a common fraction. 72. \_\_\_\_\_
73. The measure of an exterior angle of a triangle is 75 degrees. If one of the non-adjacent interior angles measures 28 degrees, what is the number of degrees in the other non-adjacent interior angle? 73. \_\_\_\_\_ (degrees)
74. If the radius of a circle is increased by 30%, by what percent is the area increased? 74. \_\_\_\_\_ (percent)
75. Winnifred has 400 wooden blocks that are exactly 1-inch cubes. What is the length, in inches, of a face of the largest cubical box that Winnifred could fill using these blocks? 75. \_\_\_\_\_ (inches)
76. A grocer sells sugar in 5-kg and 3-kg bags. Yesterday he sold 600 kg of sugar using the same number of 5-kg bags as 3-kg bags. What is the total number of bags of sugar that were sold by the grocer yesterday? 76. \_\_\_\_\_ (bags)
77. If four times  $k$  is added to 8, the result is 44. What is the value of  $k^2$ ? 77. \_\_\_\_\_
78. What is the sum of the distinct prime factors of the integer 385? 78. \_\_\_\_\_
79. If  $64^5 = 32^x$ , what is the value of  $2^{-x}$ ? Express your answer as a common fraction. 79. \_\_\_\_\_
80. Given that 10 is the arithmetic mean of the set  $\{6, 13, 18, 4, x\}$ , what is the value of  $x$ ? 80. \_\_\_\_\_