2012 Chapter Competition

Sprint Round

1. Mrs. Smith teaches for 5½ hours each day so how many hours does she teach in a 6 day period?
   6(5½) = 6(5) + 6(½) = 30 + 3 = 33 **Ans.**

2. ⌘ is defined as:
   a ⌘ $b = 6a - b$
   4 ⌘ 22 = (6 × 4) – 22 =
   24 – 22 = 2 **Ans.**

3. 7 apples cost x dollars.
   (7 × 10) or 70 apples cost (x × 10) or 10x dollars.
   10x **Ans.**

4. The results of a survey at East Elementary show that 5 families have 1 child, 10 have 2 children, 3 families have 3 children and 2 have 4 children. We are asked to find how many families have 3 or more children.
   That's 3 + 2 = 5 families.
   5 **Ans.**

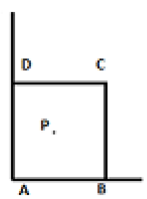
5. The total weight of 5 identical blocks is 3 lbs 12 oz. So what is the weight of one block? There are 16 oz in 1 lb, so converting the weight to ounces we get
   3 lbs 12 oz = (16 × 3) + 12 = 60
   60/5 = 12 **Ans.**

6. $\frac{4}{2x} = \frac{1}{3}$
   Cross multiplying yields
   $2x = 12$
   $x = 6$ **Ans.**

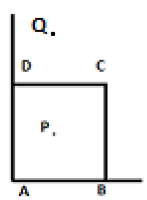
7. Maureen rented a lemonade stand for $10 per week. Lemonade is 75¢ per cup, and there are 25¢ in supplies associated with each cup of lemonade. We must find how many cups of lemonade she has to sell so she comes out even.
   Let x = the number of cups she cells.
   Let's convert all monetary values to cents.
   $1000 + 25x = 75x$
   $1000 = 50x$
   $x = 20$ **Ans.**

8. Hugo has 5 quarters, 13 dimes and 7 nickels. Luigi has 6 quarters, 10 dimes and 2 nickels. So how much more does Hugo have than Luigi?
   Hugo has (5 × 25) + (13 × 10) + (7 × 5) = 125 + 130 + 35 = 290 cents.
   Luigi has (6 × 25) + (10 × 10) + (2 × 5) = 150 + 100 + 10 = 260 cents.
   290 − 260 = 30 **Ans.**

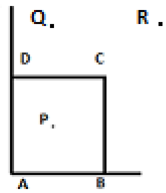
9. Square ABCD has vertices A(0, 0), B(4, 0), C(4, 4), D(0, 4).



P is in the interior of the square and is reflected over the line containing segment CD to point Q. If P is at (x, y), then Q is at (x, $4 + 4 - y$) or (x, $8 - y$).



Point Q is reflected over the line containing segment BC to point R.
R is located at ($4 + 4 - x$, $8 - y$) or ($8 - x$, $8 - y$)

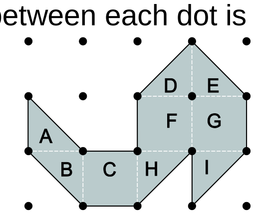


The midpoint of segment PR is
$\left(\frac{8-x+x}{2}, \frac{8-y+y}{2}\right)$ = (4, 4) **Ans.**

10. What percent of the positive integers ≤ 36 are factors of 36?
The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36. So, we have 9 factors for 36.
$\frac{9}{36} = \frac{1}{4}$ = 25% **Ans.**

11. Three questions are asked in one round of a game show. The second question is worth twice as much as the first. The third is worth three times as much as the second. The third question is worth $12,000. So what is the value of the first question?
We are told that the third question is worth 3 times as much as the second. Let x represent the value of the second question.
$3x = 12{,}000$
$x = 4000$
The second question is worth twice as much as the first. So the first question must be worth 4000 ÷ 2 = 2000 dollars.
2000 **Ans.**

12. We are asked to find the area of the shaded (grey) region where the distance between each dot is 1 cm.



The regions labeled A and B have a combined area of 1 cm^2. The region labeled C has an area of 1 cm^2. The regions labeled H and I have a combined area of 1 cm^2. The regions labeled F and G have a combined area of 2 cm^2. The regions labeled D and E have a combined area of 1 cm^2. Therefore, the grey region has an area, in square centimeters, of
4(1) + 2 = 6 **Ans.**

13. Seven consecutive positive integers have a sum of 91. So what is the largest of these integers?
If we call the middle (4th) number x, the seven consecutive integers can be represented as
$x - 3$, $x - 2$, $x - 1$, x, $x + 1$, $x + 2$ and $x + 3$.
These integers have a sum of $7x$. So we have
$7x = 91$
$x = 13$
The largest integer is $x + 3 = 13 + 3 =$ 16 **Ans.**

14. We are asked to find the largest sum of calendar dates for seven consecutive Fridays in any given year. (Note: Just because we're given the example using Friday the 13th doesn't mean we have to use it and we don't have to start at the beginning of the month either.)
We want to use the largest numbers we can. If the 31st is a Friday, then previous Fridays in the month are the 24th, 17th, 10th and 3rd. What are the next several Fridays after the 31st?
7, 14, 21, 28.
We're looking for 7 dates.
3 + 10 + 17 + 24 + 31 + 7 + 14 = 106
Let's remove 3 and 10, and add 21 and 28 to see if we get a larger sum.
17 + 24 + 31 + 7 + 14 + 21 + 28 = 142

If we remove the 17, we'll need to add a value after the 28 and that will be (no matter what month we're talking about) less than 17.
142 **Ans.**

15. A cylindrical container has a diameter of 8 cm (i.e., a radius of 4 cm) and a volume of 754 cm^2. Another container also has a diameter of 8 cm, but is twice as tall as the original container. We are asked to find the volume of the second container.
Recall that for a cylinder with height, *h*, $V = \pi r^2 h$, and in this case $\pi r^2 h = 754$.
For the second cylinder we have a new height, *H*, which is equal to 2*h*.
$\pi r^2 H = \pi r^2(2h) = 2\pi r^2 h = 2 \times 754 =$
1508 **Ans.**

16. The angles of a triangle are in the ratio 1:3:5. What is the degree measure of the largest angle in the triangle? Since the sum of the measures of the angles of a triangle is 180°, we have
$x + 3x + 5x = 180$
$9x = 180$
$x = 20$
$5x = 100$ **Ans.**

17. Four numbers are written in a row. The mean of the first two is 10 and the mean of the last two is 20. We are asked to find the mean of all four numbers.
Call the four numbers *a*, *b*, *c* and *d*.
Then we have:
$$\frac{a+b}{2} = 10$$
$a + b = 20$
and
$$\frac{c+d}{2} = 20$$
$c + d = 40$
Therefore, the mean of the 4 numbers is:
$$\frac{a+b+c+d}{4} = \frac{20+40}{4} = \frac{60}{4} = 15$$ **Ans.**

18. Carol, Jane, Kim, Nancy and Vicky competed in a 400-meter race. Nancy beat Jane by 6 seconds. Carol finished 11 seconds behind Vicky. Nancy finished 2 seconds ahead of Kim, but 3 seconds behind Vicky. We are asked to find by how many seconds Kim finished ahead of Carol.
Let *C*, *J*, *K*, *N* and *V* represent the times for Carol, Jane, Kim, Nancy and Vicky, respectively.
Nancy beat Jane by 6 seconds means that Nancy finished 6 seconds before Jane. Therefore, $N + 6 = J$.
Carol finished 11 seconds behind Vicky, which means that it took Carol 11 seconds more than Vicky, $V + 11 = C$
Nancy finished 2 seconds ahead of Kim, which means Nancy took 2 seconds longer than Kim did to finish, $N + 2 = K$
Finally, Nancy finished 3 seconds behind Vicky, which means it took Nancy 3 more seconds than Vicky to finish the race, $V + 3 = N$
We need to find *K* in terms of *C*.
The equation $V + 11 = C$ can be rewritten as $V + 3 + 8 = C$. Since $N = V + 3$, substituting yields
$N + 8 = C$. This can be rewritten as
$N + 2 + 6 = C$. Remember that
$N + 2 = K$, and substituting yields
$K + 6 = C$
Therefore, Kim finished 6 seconds before Carol. 6 **Ans.**

19. *S* and *T* are both two-digit integers less than 80. Each number is divisible by 3. *T* is also divisible by 7. *S* is a perfect square. $S + T$ is a multiple of 11, so what is the value of *T*?

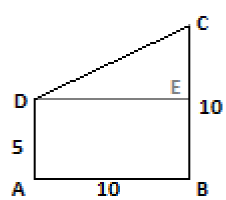
S is a perfect square whose value is less than 80. Let's list the perfect squares less than 80. They are
1, 4, 9, 16, 25, 36, 49, 64
But since S is also divisible by 3, let's remove any square that is not divisible by 3. That leaves just 9 and 36.
T is divisible by 3 and by 7 which means it must be divisible by 21. Let's list the multiples of 21 that are less than 80.
21, 42, 63
S + *T* is a multiple of 11. Let's just look for a sum that is a multiple of 11 then.
9 + {21, 42, 63} = {30, 51, 72}
None of those are multiples of 11.
36 + {21, 42, 63} = {57, 78, 99}
And 99 is a multiple of 11.
Therefore, *T* must be 63. **Ans.**

20. In a stack of six cards, each card is labeled with a different integer from 0 to 5. Two cards are selected at random without replacement. So what is the probability that their sum will be 3?
There are a total of 6 × 5 = 30 combinations for randomly selecting 2 cards without replacement. Of those, the combinations with a sum of 3 are (0, 3), (3, 0), (1, 2) and (2, 1). There are a total of 4 such combinations.
$\frac{4}{30}=\frac{2}{15}$ **Ans.**

21. Okta stays in the sun for 16 minutes before getting sunburned. Using a sunscreen, he can stay in the sun 20 times as long before getting sunburned (or 320 minutes). If he stays in the sun for 9 minutes and then applies the sunscreen, how much longer can he remain in the sun? Okta's already used 9 of his 16 minutes, and has a remainder of 16 – 9 = 7 minutes before getting sunburned. Since the sunscreen lets him stay in the sun 20 times longer, using the sunscreen will give him an additional 7 × 20 = 140 minutes.
140 **Ans.**

22. A bag contains ten each of red and yellow balls. The balls of each color are numbered from 1 to 10. If two balls are drawn at random, without replacement, then what is the probability that the yellow ball numbered 3 is drawn followed by a red ball?
The probability that the yellow ball numbered 3 is drawn is 1/20.
Now there are 19 balls left. The probability that a red ball is drawn is 10/19.
So, the total probability is:
$\frac{1}{20}\times\frac{10}{19}=\frac{1}{38}$ **Ans.**

23. In trapezoid ABCD, AB = BC = 2AD and AD = 5. We are asked to find the area of trapezoid ABCD.



AB and BC are both 10. Draw segment DE, parallel to segment AB intersecting segment BC at point E. Then ABED is a rectangle whose area is 10 × 5 = 50. That leaves triangle DEC which is a right triangle with legs 10 and 5. The area of this triangle is
½ × 10 × 5 = 25. The total area of trapezoid ABCD is 50 + 25 = 75 **Ans.**

24. One line has a slope of −1/3 and contains the point (3, 6). Another line has a slope of 5/3 and contains the point (3, 0). We are asked to find the product of the coordinates of the point at which

the two lines intersect.
Let's determine the equation in the form $y = mx + b$ of the first line, where $m = -1/3$, $x = 3$ and $y = 6$.
$6 = (-1/3) \times 3 + b$
$6 = -1 + b$
$b = 7$
So the equation of the first line is
$y = (-1/3)x + 7$
Now, for the second equation, with $m = 5/3$, $x = 3$ and $y = 0$, we have
$0 = (5/3) \times 3 + b$
$0 = 5 + b$
$b = -5$
So the equation for the second line is
$y = (5/3)x - 5$
We must find the coordinates of the point at which the two lines intersect.
$(-1/3)x + 7 = (5/3)x - 5$
$-x + 21 = 5x - 15$
$6x - 15 = 21$
$6x = 36$
$x = 6$
Now let's get the value of y when $x = 6$.
$y = (5/3)x - 5$
$y = (5/3) \times 6 - 5 = 10 - 5 = 5$
(If we had used the first equation we'd get the same value for y.)
Thus, the coordinates of the point at which the two lines meet are (6, 5). The product of the two coordinates is:
$6 \times 5 = 30$ **Ans.**

25. A car traveled a certain distance at 20 mph. Then it traveled twice the distance at 40 mph. The entire trip lasted for four hours and we must find the total number of miles driven.
Let x = the distance traveled at 20 mph.
Then $2x$ = the distance traveled at 40 mph.
Let y = the time that was traveled at 20 mph. Then $y = x/20$.
The number of hours traveled at 40 mph was $2x/40 = x/20 = y$.
So the car traveled 2 hours at 20 mph and 2 hours at 40 mph.
2 hours at 20 mph is 40 miles.
2 hours at 40 mph is 80 miles.
40 + 80 = 120 **Ans.**

26. Consider all integer values of a and b for which $a < 2$ and $b \geq -2$.
We are asked to find the minimum value of $b - a$.
To minimize the value of $b - a$, we need to consider the least possible value for b and the greatest possible value for a. So we have $-2 - 1 = -3$ **Ans.**

27. In the figure shown, the diagonals of a square are drawn and then two additional segments from each vertex to a diagonal. How many triangles are in the figure?
The triangles are identified in the following 12 figures:

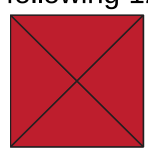
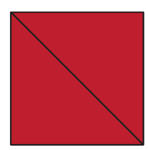
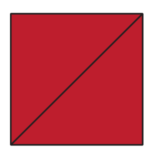


Figure 1 shows 4 congruent triangles



Figures 2 shows 2 congruent triangles



Figures 3 shows 2 congruent triangles

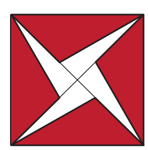


Figure 4 shows 4 congruent triangles

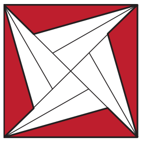


Figure 5 shows 4 congruent triangles

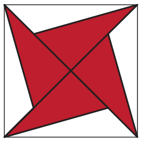


Figure 6 shows 4 congruent triangles

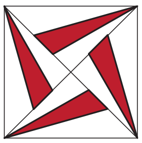


Figure 7 shows 4 congruent triangles

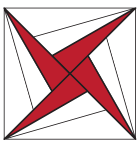


Figure 8 shows 4 congruent triangles

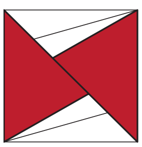


Figure 9 shows 2 congruent triangles

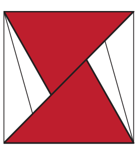


Figure 10 shows 2 congruent triangles

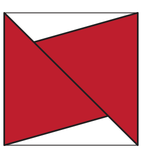


Figure 11 shows 2congruent triangles

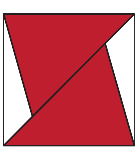
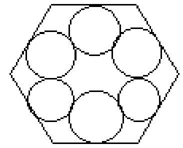


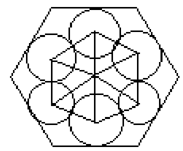
Figure 12 shows 2 congruent triangles

The total number of triangles is 36 **Ans.**

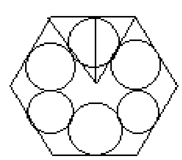
28. Six circles of radius, $r = 1$ unit are drawn in the hexagon as shown.



We must find the perimeter of the hexagon. Draw two lines from the center of the hexagon to the centers of each of the circles. and connect each center to the adjacent one to form triangles.



The 6 triangles created are congruent equilateral triangles. (Each one has a 60° angle at the center of the hexagon and both segments connecting the point at the center of the hexagon to the center.)
The base of each triangle is 2. This means that the length from the center of the hexagon to the center of the circle is also 2 (equilateral triangle).
If we now extend the line to the edge of the hexagon we know that the length from the center of the hexagon to the center of the side of the hexagon is $2 + r = 2 + 1 = 3$.
Now we draw a triangle from the center of the hexagon to the two ends of a side of the hexagon. We also draw the height (which we know is 3).



Let s = the side of the hexagon.
We know the height, $h = 3$ and it is

opposite the 60° angle. Therefore

$$\frac{\sqrt{3}}{2}s = 3$$

$$\sqrt{3}s = 6$$

$$s = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

The perimeter is

$6s = 6 \times 2\sqrt{3} = 12\sqrt{3}$ **Ans.**

29. If $x^2 + \frac{1}{x^2}$ = 3, then what is the value of $\frac{x^2}{(x^2+1)^2}$?

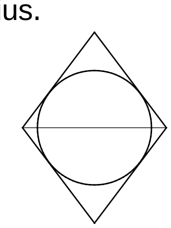
Let's take $x^2 + \frac{1}{x^2}$ = 3 and multiply both sides by x^2 to remove the fraction.
We get $x^4 + 1 = 3x^2$.
Now expand the second expression.

$$\frac{x^2}{(x^2+1)^2} = \frac{x^2}{x^4 + 2x^2 + 1}$$

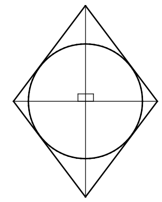
Substituting for $x^4 + 1$ we get:

$$\frac{x^2}{3x^2 + 2x^2} = \frac{x^2}{5x^2} = \frac{1}{5}$$ **Ans.**

30. A circle is inscribed in a rhombus with sides of length 4cm. The two acute angles each measure 60°. We are asked to find the length of the circle's radius.

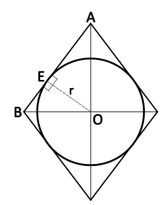


Since the sum of the measures of the angles of a quadrilateral is 360°, we know that the two other angles in the rhombus each measure [360 – 2(60)]/2 = (360 – 120)/2 = 240/2 = 120°. Draw the shorter diagonal through the rhombus and you've created an equilateral triangle so the smaller diagonal is also of length 4. Now draw the other diagonal.



The two diagonals intersect at a 90° angle creating 4 congruent 30-60-90 right triangles, each with a hypotenuse of length 4 cm. By the properties of 30-60-90 right triangles we have that the longer leg of each triangle has length $2\sqrt{3}$ cm.

Draw a line from the center of the circle to the point of tangency between the circle and the rhombus.



The line OE is perpendicular to AB. That makes triangle BEO a 30-60-90 right triangle, Triangle ABO is also 30-60-90 right triangle. Using similar triangles:

$$\frac{r}{2\sqrt{3}} = \frac{2}{4} =$$

$4r = 4\sqrt{3}$

$r = \sqrt{3}$ **Ans.**

Target Round

1. Tawana purchases 3 CDs for $6.98, $7.49 and $15.63. If the three prices sum to less than $30, then shipping is $3; otherwise it's 10% of the total price. To determine the total cost of Tawana's merchandise, we add 6.98 + 7.49 + 15.63 = 30.10. Her merchandise totals $30.10 which is more than $30. So we have to compute 10% of 30.10 for

shipping.
30.10 × 0.1 = 3.01
30.10 + 3.01 = 33.11 **Ans.**

2. Three and one half hours ago it was 10:15 am. That means it is now 1:45 pm. We must find how many more minutes it is from now until the next noon. Once we get past midnight we'll have 12 hours. Then 1:45 pm to midnight is 10 hours and 15 minutes. That's a total of 22 hours and 15 minutes.
(22 × 60) + 15 = 1320 + 15 = 1335 **Ans.**

3. A line containing the points (–8, 9) and (–12, 12) intersects the *x*-axis at point P. We must find the *x*-coordinate of point P.
Using the two points, let's find the equation of the line.
$9 = -8m + b$ (using the first point)
$12 = -12m + b$ (using the second point)
$-3 = 4m$
$m = -3/4$
Substituting back into the first equation:
$9 = (-3/4)(-8) + b$
$9 = 6 + b$
$b = 3$
Now, we can write the equation of the line.
$y = (-3/4)x + 3$
The line crosses the *x*-axis when $y = 0$.
$0 = (-3/4)x + 3$
$(3/4)x = 3$
$x = 12/3 = 4$ **Ans.**

4. Mike wrote a list of 6 positive integers on his paper. The first two are chosen randomly. Each of the remaining integers is the sum of the two previous integers. We are asked to find the ratio of the fifth integer to the sum of all 6 integers.
Let x_1 and x_2 be the first two integers.
$x_3 = x_1 + x_2$
$x_4 = x_2 + x_3 = x_2 + x_1 + x_2 = x_1 + 2x_2$
$x_5 = x_3 + x_4 = x_1 + x_2 + x_1 + 2x_2$
$x_5 = 2x_1 + 3x_2$
$x_6 = x_4 + x_5 = x_1 + 2x_2 + 2x_1 + 3x_2$
$x_6 = 3x_1 + 5x_2$
We have the expression for the 5th integer. So let's get the sum of all 6.
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = x_1 + x_2 + x_1 + x_2 + x_1 + 2x_2 + 2x_1 + 3x_2 + 3x_1 + 5x_2 = 8x_1 + 12x_2$
The requested ratio is:
$$\frac{2x_1+3x_2}{8x_1+12x_2}=\frac{2x_1+3x_2}{4(2x_1+3x_2)}=\frac{1}{4}$$ **Ans.**

5. Sonia has 5 more pairs of shoes than Danielle. Imelda has twice as many pairs as Sonia. The girls have a total of 39 pairs of shoes. We are asked to find how many more pairs of shoes Imelda has than Sonia and Danielle combined.
Let *S*, *D* and *I* be the number of shoes that Sonia, Danielle and Imelda have, respectively. Then:
$S = D + 5$ (first sentence)
$I = 2S$ (second sentence)
$S + D + I = 39$ (third sentence)
Let's get *I* in terms of *D*.
$I = 2S = 2 \times (D + 5) = 2D + 10$
$S + D + I = D + 5 + D + 2D + 10 = 39$
$4D + 15 = 39$
$4D = 24$
$D = 6$
$S = 6 + 5 = 11$
$I = 2S = 22$
$S + D = 11 + 6 = 17$
$I - (S + D) = 22 - 17 = 5$ **Ans.**

6. How many positive integers ≤ 2000 have an odd number of factors?
Positive integers that have an even number of factors are not squares.
Why? Primes always have 2 factors (1

and the number itself). Other integers that are not squares will have factors like this:

$f_1 \times f_2$

$f_3 \times f_4$

…

$f_{n-1} \times f_n$

Squares have similar factors, **but** they have one more (assume x is a square):

$\sqrt{x} \times \sqrt{x}$

So we need to only figure out how many squares there are ≤ 2000.

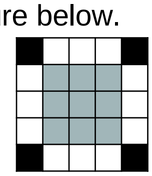
44 × 44 = 1936

45 × 45 = 2025

44 **Ans.**

7. A 5 × 5 × 5 cube is painted on 5 of its 6 faces. It is then cut into 125 unit cubes. One unit cube is randomly selected and rolled. We are asked to find the probability that the top face of the cube that is rolled is painted.

Assume that the top of the cube, as it sits on a surface, is the one face that is not painted. Then, of the 25 cubes on the top level, we have 9 (in grey) that have no sides painted, 12 (in white) that have one side painted and 4 (in black) that have two sides painted as shown in the figure below.



The same holds for the second, third and fourth level.

On the fifth level, which is the bottom, all cubes have their bottom painted and some have some of their sides painted (but not their tops). In particular, using the same image above 4 cubes have 3 sides painted (in red), 12 (in green) have 2 sides painted and 9 (in blue) have 1 side painted.

Let's total everything up.

The number of cubes that have 0 sides painted is (9 × 4) = 36.

The number of cubes that have only 1 side painted is (4 × 12) + 9 = 57.

The number of cubes that have 2 sides painted is (4 × 4) + 12 = 28.

The number of cubes that have 3 sides painted is 4.

Let's just do a check here.

36 + 57 + 28 + 4 = 125

Okay, we're good.

What is the probability that we choose a cube with no sides painted? 36/125

What is the probability that we choose a cube with 1 side painted? 57/125

What is the probability that we choose a cube with 2 sides painted? 28/125

What is the probability that we choose a cube with 3 sides painted? 4/125

Now let's look at probabilities that a painted side will come up when the cube is rolled.

What is the probability that a cube with no sides painted will have a painted side on the top when rolled? 0

What is the probability that a cube with 1 side painted will have a painted side on the top when rolled? 1/6

What is the probability that a cube with 2 sides painted will have a painted side on the top when rolled? 2/6

What is the probability that a cube with 3 sides painted will have a painted side on the top when rolled? 3/6

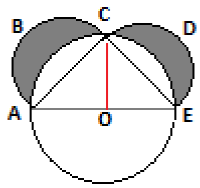
Finally, let's put it all together.

The probability that one randomly chosen cube is rolled and the top face is painted is:

$$\frac{36}{125} \times 0 + \frac{57}{125} \times \frac{1}{6} + \frac{28}{125} \times \frac{2}{6} + \frac{4}{125} \times \frac{3}{6} =$$

$$\frac{57+56+12}{125 \times 6} = \frac{125}{125 \times 6} = \frac{1}{6}$$ **Ans.**

8. Circle O has diameter AE and AE = 8. Point C is on the circumference of the circle such that segments AC and CE are congruent. Segment AC is a diameter of semicircle ABC and segment CE is a diameter semicircle CDE. What is the total combined area of the shaded regions?



Draw a line through points A and E to point C to create the triangle ACE. Also, draw the altitude from point C to point O. Both segments AO, CO and EO are radii of the circle and CO is perpendicular to diameter AE. That makes triangle ACE a 45-45-90 right triangle and AO = CO = 4.
By the properties of 45-45-90 right triangles, the length of segment AC is $4\sqrt{2}$.
So $4\sqrt{2}$ is the diameter of semicircle ABC (and also of semicircle CDE) making the radius of the semicircle $2\sqrt{2}$. That's actually the radius of both semicircles so let's just get the area of a complete circle with that radius.
$A = \pi r^2 = \pi(2\sqrt{2})^2 = 8\pi$
But we still have to remove the area bounded by the hypotenuse of a triangle and the circumference of the circle. To do that let's determine the area of the quarter-circle defined by AOC. That is 1/4 of the area of the entire circle whose radius is 4.
$A = (1/4)\pi r^2 = (1/4)\pi 4^2 = (1/4)16\pi = 4\pi$
If we now subtract from that the area of the triangle AOC, we'll get the area of one of the shaded portions.
The area of the triangle is (1/2) × 4 × 4 = 8. So, the area of the quarter circle minus the area of the triangle is 4π − 8. This is the amount we have to subtract from the area of the semicircle but remember that we have 2 of these or 8π – 16. Subtracting this amount from the area of the entire circle we get
8π – (8π – 16) = 16 **Ans.**

Team Round

1. The first third of tickets for the play sell for \$8 each. Remaining tickets sell for \$10 each. There are 27 rows of seats with 44 seats in each row. We are asked to find out how much will be collected from selling all the tickets.
First let's figure out how many seats are available.
27 × 44 = 1188 seats
The first third sell for \$8 each.
1188 ÷ 3 = 396
396 × 8 = 3168
That's a total of \$3168. The remaining 1188 – 396 = 792 tickets sell for \$10 each. That's a total of \$7920.
7920 + 3168 = 11,088 **Ans.**

2. The endpoints of a diameter of a circle are (–1, –4) and (–7, 6). We must find the coordinates of the center of the circle, which is also the midpoint of this line segment. Start with the *x*-coordinates of the segment:
−7 – (−1) = −7 + 1 = −6
Half of –6 is –3.
−1 + (−3) = −4
This is the x-coordinate of the midpoint.
Now let's work on the *y*-coordinate.
6 – (−4) = 6 + 4 = 10
Half of 10 is 5.
−4 + 5 = 1
This is the *y*-coordinate of the midpoint.

The ordered pair for the center of the circle is (–4, 1) **Ans.**

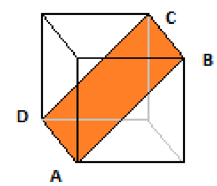
3. The mean of {1, 2, 4, 8, 9, 10, 14, 16, 17} is 9. If one number is removed and the mean decreases by 1, what is the value of the number that was removed?
Let x represent the number that is removed. The sum of all 9 numbers since the mean is 81. If the mean of 8 numbers is 8, the sum of the 8 numbers must be 64, since 64/8 = 8. To find the number that was removed we subtract
$81 - x = 64$
$x = 17$ **Ans.**

4. Line l is perpendicular to the line with equation $6y = kx + 24$. The slope of line l is –2. We must find the value of k.
Let's rewrite the equation in the form
$y = mx + b$.
$6y = kx + 24$
$y = (k/6)x + 4$
The slope of a perpendicular line to line l is the negative reciprocal of the slope of line l.
$-(-2)^{-1} = 1/2$
So $\frac{k}{6} = \frac{1}{2}$ and $k = 3$ **Ans.**

5. Sam's monthly commission is
$C = 270g + 3g^2$, where g is the number of cars that Sam sells. Sam sold 30 cars so how much commission did Sam earn?
$C = 270g + 3g^2 = 270(30) + 3(30)^2 =$
$8100 + 2700 = 10{,}800$ **Ans.**

6. There is a shallow fish pond in the shape of a square. The perimeter of the pond is 24 ft and the water is 6 in deep. We must find the volume of the water in the pond.
First thing to note is that the water depth is in **inches** not **feet.** 6 in is ½ foot.
Now, let's find the side of the fish pond.
$4s = 24$
$s = 6$
The volume of the pond is, therefore,
$6 \times 6 \times (1/2) = 18$ **Ans.**

7. The cube shown has a side length of s. Points A, B, C and D are vertices of the cube. We need to find the area of rectangle ABCD.



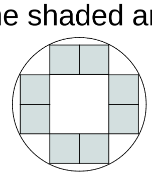
We need to find the length of AB (or CD) and BC (or AD).
AB is just the diagonal of a square of side s so the length is $s\sqrt{2}$. BC is the side of the square so it's just s.
The area of the rectangle is $s \times s\sqrt{2} =$
$s^2\sqrt{2}$ **Ans.**

8. It rained on exactly 10 days during Tricia's vacation. It rained either in the morning or in the afternoon on each rainy day. There were 13 mornings when it didn't rain and 17 afternoons when it didn't rain. So how many days did Tricia's vacation last?
Let x represent the number of days it rained in the morning. Let y represent the number of days it rained in the afternoon. Let z represent the number of days it didn't rain at all.
$x + y = 10$
$x + z = 17$ (no rain in the afternoon)
$y + z = 13$ (no rain in the morning)
Subtracting these two equations we get $x - y = 4$. Now we have $x + y = 10$ and $x - y = 4$. Adding these two equations we get
$2x = 14$

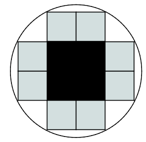
$x = 7$
So $7 + y = 10$ and $y = 3$. We then have $3 + z = 13$ and $z = 10$. That means $x + y + z = 7 + 3 + 10 = 20$ **Ans.**

9. If the letters of the word ELEMENT are randomly arranged, what is the probability that the three E's are consecutive?
Other than the E's, we have L, M, N, and T.
If three E's are consecutive we can have:
EEEabcd
aEEEbcd
abEEEcd
abcEEEd
abcdEEE
where a is the first choice from the letters L, M, N, T and b is the second choice etc.
That's 5 different placements for three E's together.
In each placement, the 4 non-E's can appear in 4! = 24 combinations. The 3 E's can appear in 3! = 6 combinations.
$$\frac{5\times24\times6}{7!}=\frac{5\times24\times6}{7\times6\times5\times4\times3\times2\times1}=$$
$$\frac{24}{7\times4\times3\times2\times1}=\frac{1}{7}$$ **Ans.**

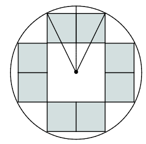
10. The diagram shows 8 congruent squares inside a circle. Find the ratio of the shaded area to the area of the circle.



Let s represent the side of one of the squares. Then the area of shaded regions is $8s^2$. The area of the middle square (in black) is $2s \times 2s = 4s^2$. It means that the distance from the center of the circle to the point at which 2 grey squares meet is also s.



Draw a triangle from the center of the circle to the points of the two s by s shaded area and also draw the height.



The 2 identical sides of the triangle are each r. The height is $2s$ and the third side is $2s$. We can now find the value of r.
$(2s)^2 + s^2 = r^2$
$5s^2 = r^2$
Now we can work on the ratio. The area of the circle is $\pi r^2 = 5\pi s^2$. The area of the 4 shaded regions is $8s^2$. The ratio of the shaded region to the area of the circle is
$$\frac{8s^2}{5\pi s^2}=\frac{8}{5\pi}\approx 0.509296 \approx 0.51$$ **Ans.**