MATHCOUNTS

2013 Chapter Competition Solutions

Are you wondering how we could have possibly thought that a Mathlete® would be able to answer a particular Sprint Round problem without a calculator?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Target Round problem in less 3 minutes?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Team Round problem with less that 10 sheets of scratch paper?

The following pages provide solutions to the Sprint, Target and Team Rounds of the 2013 MATHCOUNTS® Chapter Competition. Though these solutions provide creative and concise ways of solving the problems from the competition, there are certainly numerous other solutions that also lead to the correct answer, and may even be more creative or more concise! We encourage you to find numerous solutions and representations for these MATHCOUNTS problems.

Special thanks to volunteer author Mady Bauer for sharing these solutions with us and the rest of the MATHCOUNTS community!

2013 Chapter Competition

Sprint Round

1. Marti, who lives in New York, calls Kathy, who lives in Honolulu. Marti calls at 6:30 p.m. in New York.
   The chart shows that when it is noon in New York it is 7 AM in Honolulu. That means that Honolulu is 5 hours earlier than New York.
   6:30 p.m. – 5 = 1:30 p.m. **Ans.**

2. What is the value of 1 – 2 + 4 – 8 + 16 – 32 + 64 – 128 + 256 – 512 +1024?
   1 + (– 2 + 4) + (– 8 + 16) + (– 32 + 64) + (– 128 + 256) + (– 512 + 1024) =
   1 + 2 + 8 + 32 + 128 + 512 = 683 **Ans.**

3. 690 names are read at a rate of 1 name every 10 seconds.
   This means that 60 ÷ 10 = 6 names are read every minute. Therefore, it will take 690 ÷ 6 = 115 minutes. 115 **Ans.**

4. According to the bar graph, Ms. Pinski's class collected 9 red apples, 3 yellow apples and 8 green apples.
   There are 9 – 3 = 6 more red apples than yellow apples. 6 **Ans.**

5. The perimeter of a rectangle is 18 cm. The length of the rectangle is one-third of its perimeter so what is the width?
   Let l represent the length of the rectangle and w represent its width. We have $l = (1/3) \times 18 = 6$. Using the perimeter formula, we have
   $2l + 2w = 18$
   $(2 \times 6) + 2w = 18$
   $12 + 2w = 18$
   $2w = 6$
   $w = 3$ **Ans.**

6. One-half of the sum of n and 8 is 7. Find n. Let's rewrite this as an equation and solve:
   $\frac{1}{2}(n + 8) = 7$
   $n + 8 = 14$
   $n = 6$ **Ans.**

7. When (37 × 45) – 15 is simplified, what is the units digit?
   Consider the units digits of 37 and 45. Since 7 × 5 = 35, the units digit of 37 × 45 is 5. Since 15 also has a units digit of 5, subtracting results in a units digit of 0. **Ans.**

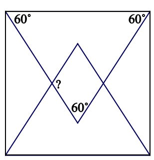
8. One witness says the suspect is 25 years old and 69 inches tall. A second witness says the suspect is 35 years old and 74 inches tall. And a third witness says the suspect is 35 years old and 65 inches tall. Each witness correctly identified the suspect's age or height but not both. Let a represent the suspect's age and b represent the suspect's height. We must find $a + b$.
   Let's assume the first witness identified the suspect's correct age, 25 years old. That means the other two witnesses were wrong about the age, so they must have correctly identified the suspect's height. But they gave two different values for the height. So, the first witness must have correctly identified the suspect's height of 69 inches, and the suspect is 35 years old. Therefore, $a = 35$, $b = 69$ and $a + b = 104$ **Ans.**

9. The rabbit jumps halfway to a carrot. If the rabbit lands within 6 inches of the carrot he will eat it. If the rabbit is originally 12 feet away, how many times must the rabbit jump in order to eat the carrot?
   After the first jump, the rabbit's distance

from the carrot is 6 feet. After the second jump, it’s 3 feet. After the third jump, it’s 1.5 feet, or 18 inches. After the fourth jump, it’s 9 inches. After the fifth jump, it’s 4.5 inches, which means the rabbit has a rather tasty carrot. 5 **Ans.**

10. Mellie has 6 pairs of pants and 10 shirts. She buys 2 more pairs of pants. How many more outfits can Mellie make now?
Each new pair of pants can be paired with each of the 10 shirts, to make a total of 2 × 10 = 20 additional outfits.
20 **Ans.**

11. Two equilateral triangles are drawn in a square. Find the measure of each obtuse angle in the rhombus formed by the intersection of the triangles.



The rhombus formed by the intersection of the triangles has two opposite angles of 60°. The other two angles are both the same size.
Let x represent the measure of one of the two unknown angles in the rhombus. Since the sum of the interior angles of a quadrilateral is 360°, we have
$2x + 60 + 60 = 360$
$2x + 120 = 240$
$2x = 240$
$x = 120$ **Ans.**

12. For 7 lbs of Mystery Meat and 4 lbs of Tastes Like Chicken the cost is $78. Tastes Like Chicken costs $3 more per pound than Mystery Meat. So how much does a pound of Mystery Meat cost?
Let m represent the cost per pound of Mystery Meat, and c represent the cost per pound of Tastes Like Chicken. We have two equations: $7m + 4c = 78$ and $c = m + 3$. Substitute and solve to get
$7m + 4(m + 3) = 78$
$7m + 4m + 12 = 78$
$11m = 66$
$m = 6$ **Ans.**

13. The perimeter of a rectangle is 22 cm. The area is 24 cm^2. What is the smaller of the two integer dimensions of the rectangle?
Let l represent the length of the rectangle and w represent its width. Then $2(l + w) = 22$ and $l + w = 11$. We also know $lw = 24$.
The factors of 24 are:
1 × 24
2 × 12
3 × 8
4 × 6
Of these, only 3 and 8 add to 11. The smaller of the two integer dimensions must be 3. **Ans.**

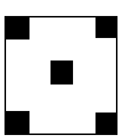
14. Mr. Cansetti’s home is 7.5 blocks from the police station. The post office is 6 blocks from the grocery store and 3.5 blocks from the police station. The order in which the buildings are is Mr. Cansetti’s home, the post office, the police station and then the grocery store. How far is it from his home to the store?
The post office is 3.5 blocks from the police station. Therefore, the police station is 6 – 3.5 = 2.5 blocks from the grocery store. From Mr. Cansetti’s home to the post office is 7.5 – 3.5 = 4 blocks. Now we can add it all up: 4 blocks (home to post office) + 3.5 blocks (post office to police) + 2.5 blocks (police to

grocery) is 10 blocks. 10 **Ans.**

15. Hank has less than 100 pigs. 5 pigs to a pen results in 3 pigs left over. 7 pigs to a pen results in 1 pig left over. 3 pigs to a pen results in 0 pigs left over.
Let x represent Hank's number of pigs. We know that x is divisible by 3 and ends in 3 or 8 since there is a remainder of 3 when x is divided by 5. Given that when x divided by 7 leaves a remainder of 1, we must be looking for a number ending in 2 or 7 that is divisible by 7. Those numbers less than 100 are 7, 42, and 77. Add 1 to each and choose the one that is divisible by 3. That must be 77. We have 77 + 1 = 78. Checking, we see that 78 is divisible by 3, 78 ÷ 7 = 11r1 and 78 ÷ 5 = 15r3. 78 **Ans.**

16. 3 people can paint 5 rooms in 2 days. How long does it take 6 people to paint 15 rooms.
It follows that 6 people can paint 10 rooms in 2 days, and 6 people can paint 5 rooms in 1 day. Therefore, 6 people can paint 15 rooms in 3 days. 3 **Ans.**

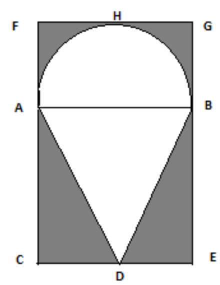
17. A square dartboard has 5 smaller shaded squares. The side of dartboard is 4 times the side of a shaded square. What's the probability that a dart lands in a shaded area?



Let x represent the length of a shaded area. Then x^2 is the area of one of the shaded squares and $5x^2$ is the entire shaded area. The length of the side of the dartboard, then, is $4x$, which means that the area of the dartboard is $16x^2$. The probability of landing in a shaded area is $5x^2/16x^2 = 5/16$ **Ans.**

18. A line passes through the points (−2, 8) and (5, −13). When the equation of the line is written in the form $y = mx + b$, what is the product of m and b.
Recall that $m = (y_2 – y_1)/ (x_2 – x_1)$. So, we have $m = (8 – (–13))/(–2 – 5) = 21/(−7) = −3$. Now substituting $y = 8$, $x = −2$ and $m = −3$ into $y = mx + b$, we have
$8 = −3(−2) + b$
$8 = 6 + b$
$b = 2$
$m \times b = −3 \times 2 = −6$ **Ans.**

19. A sign has a shape consisting of a semicircle and an isosceles triangle. The rectangle measures 2 ft by 4 ft. The shaded regions will be removed. BE = 3BG and AB is parallel to CE. Find the area of the resulting region.

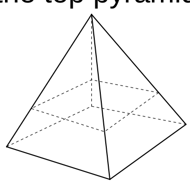


Since FG = 2 and FH = HG = 1, it follows that the semicircle with diameter AB has a radius of 1 and area π/2.
GB is the same length as the radius so it is also 1. Therefore, EB = 4 – 1 = 3, and the area of ΔABD is ½ × 2 × 3 = 3.
Thus, the total area of the isosceles triangle and the semicircle is: 3 + π/2 = 3 + 1.570795 ≈ 4.6 **Ans.**

20. A license plate has three letters followed by 3 digits. The first two letters must be consonants, excluding Y. How many different license plates are there?
There are 26 letters in the alphabet.

There are 5 vowels in the alphabet and we can't use Y.
There are 26 – 6 = 20 choices for the first two letters and 26 choices for the third letter. There are 10 choices for each of the three digits. Therefore, the number of license plates is
$20 \times 20 \times 26 \times 10 \times 10 \times 10 =$
10,400,000 **Ans.**

21. A right square pyramid has a base with a perimeter of 36 cm and a height of 12 cm. One-third of the distance from the base, the pyramid is cut by a plane parallel to its base. What is the volume of the top pyramid?



The volume of a square pyramid is $(1/3)Bh$, where B is the area of the base. With a perimeter of 36, each side of the base is 9. Since the plane cuts the pyramid at 1/3 of the distance from the base to the apex, it means that the height of the top pyramids is 2/3 the height of the larger pyramid, or $(2/3) \times 12 = 8$. That also means the length of each side of the base of the top pyramid is $(2/3) \times 9 = 6$. Therefore, the volume of the top pyramid is
$(1/3) \times 6 \times 6 \times 8 = 96$ **Ans.**

22. Four integers are chosen from 1 to 10, with repetition allowed. What is the greatest possible difference between the mean and median?
To minimize the median we choose 1, 1, 1, 10. Then the median is 1, and the mean is 13/4. The difference is
$(13/4) - (4/4) = 9/4$ **Ans.**

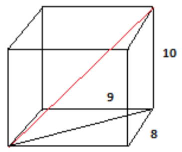
23. If Jay takes one of Mike's books, then he will be carrying twice as many books as Mike. But if Mike takes one of Jay's books, they will each be carrying the same number. We need to find out how many books Mike is carrying.
Let m represent the number of books Mike is carrying, j represent the number of books Jay is carrying. We have
$j + 1 = 2(m - 1)$
$j + 1 = 2m - 2$
$j = 2m - 3$
When Mike takes one of Jay's books:
$m + 1 = j - 1$
$m = j - 2$
$m = (2m - 3) - 2 = 2m - 5$
$m = 5$ **Ans.**

24. A rectangular prism has a volume of 720 cm^3. Its surface area is 484 cm^2 and all edge lengths are integers. We must determine what the longest segment is that can be drawn to connect two vertices.
Let w, h and l represent the prism's width, height and length, respectively.
We have $lwh = 720$ and
$2(lw + lh + wh) = 484$
$lw + lh + wh = 242$
Let's factor 720 so that we can determine what l, w and h are.
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$720 = 8 \times 9 \times 10$
Let's check the surface area:
$(8 \times 9) + (8 \times 10) + (9 \times 10) =$
$72 + 80 + 90 = 242$
So we have a rectangular prism similar to the one shown here:

B

C

A



AB is the length of the longest segment connecting two vertices. Notice that triangle ABC is a right triangle. Segment AC is also the hypotenuse of a right triangle with sides of length 8 and 9. So $(AC)^2 = 8^2 + 9^2 = 64 + 81 = 145$. It follows that
$(AB)^2 = (AC)^2 + 10^2$
$(AB)^2 = 145 + 100$
$(AB)^2 = 245$
AB = $\sqrt{245}$ = $7\sqrt{5}$ **Ans.**

25. Avi and Hari agree to meet between 5:00 p.m. and 6:00 p.m. The first person who arrives will wait for the other for only 15 minutes. Find the probability that the two of them actually meet.
If Avi gets there between 5:00 and 5:45, Hari still has a 15 minute window to arrive. The probability that Avi gets there between 5:00 and 5:45 is 3/4. The probability that Hari arrives in the next 15 minute window is 1/4. So the probability that Avi gets there in the first 45 minutes and Hari gets there within the next 15 minutes is (3/4) × (1/4) = 3/16. Similarly, the probability that Hari gets there between 5:00 and 5:45 and Avi arrives within the next 15 minutes is 3/16. This leaves dealing with the last 15 minutes. There is a probability of (1/4) × (1/4) = 1/16 that both Avi and Hari arrive during those last 15 minutes.
(3/16) + (3/16) + (1/16) = 7/16 **Ans.**

26. Old sodas were replaced by new sodas that were 20% larger. The price of the new soda is 20% less than the price of the old soda. What is the ratio of the cost per ounce of the old soda to the new soda?
Let *x* represent the number of ounces that were in the old soda, and *y* represent the price of the old soda.
The cost per ounce of the old soda is *y*/*x*. At a volume that is 20% larger, the new sodas are (6/5)*x* ounces. The price of the new soda is (4/5)*y*. The cost per ounce for the new soda is (4/5)*y*]/[(6/5)*x*] = (4/6) × (*y*/*x*) = (2/3) × (*y*/*x*).
Therefore, the ratio of the cost per ounce of the old soda to the new soda is (*y*/*x*)/[(2/3) × (*y*/*x*)] = 1/(2/3) = 3/2 **Ans.**

27. Alana and Bob can complete a job in 2 hours. Bob and Cody can do the job in 3 hours. Alana and Cody can do the same job in 4 hours. How many hours will it take all three working together to complete the job?
Since Alana and Bob can complete a job in 2 hours, they can complete 1/2 the job in one hour. Body and Cody can do the job in 3 hours. So, they can complete 1/3 of the job in one hour.
Since Alana and Cody can do the same job in 4 hours they can complete 1/4 of the job in 1 hour.
This means that 2 sets of Alana, Bob and Cody could do (1/2) + (1/3) + (1/4) = (6/12) + (4/12) + (3/12) = 13/12 of the job in one hour. One set of Alana, Bob and Cody can do (13/12) × (1/2) = 13/24 of the job in one hour. It will take them 1/(13/24) = 24/13 hours. 24/13 **Ans.**

28. What fraction of the first 100 triangular numbers is evenly divisible by 7?
The *n*th triangular number is the sum of 1 through *n*. So let's look at the first several and see if we can see a pattern.
1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66,

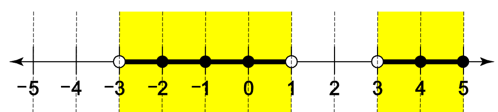
78, 91,105, 120, 136, 153, 171, 190, 210, 221
Looking at this we can see that the 6th and 7th triangular numbers, 21 and 28, are divisible by 7. The 13th and 14th triangular numbers, 105 and 120, are also divisible by 7, as are the 20th and 21st triangular numbers, 210 and 221. We have a pattern. There are 2 triangular numbers divisible by 7 in every 7 numbers. There are 100/7 ≈ 14 sets of 7 numbers. In each set we have 2 and they are always the last two of the 7 numbers. Therefore, we don't have to worry about the 99th and 100th triangular numbers. They will not be divisible by 7. That's 14 × 2 = 28 out of 100, which is 7/25 **Ans.**

29. Point A of a clock is at the tip of the minute hand. Point B is at the tip of the hour hand. Point A is twice as far from the center of the clock as point B. We must find the ratio of the distance that point B travels in 3 hours to the distance that point A travels in 9 hours.
The distance that the hour hand travels when it makes a complete revolution is $2\pi r$, where r is the distance between the tip of the hour hand and the center of the clock. The distance that the minute hand travels when it makes a complete revolution is $4\pi r$. The minute hand of a clock travels around the whole clock once per hour or one entire revolution. So point A travels $9 \times 4\pi r = 36\pi r$ in the 9 hours.
Point B travels only 1/12 of the way around the clock per hour or 360/12 = 30°. So, in 3 hours it travels 90° or 1/4 of a revolution, which is $(1/4) \times 2\pi r = (1/2)\pi r$. The ratio we must find is: $((1/2)\pi r)/(36\pi r) = 1/72$ **Ans.**

30. What percent of the interval with endpoints −5 and 5 consists of real numbers x satisfying the inequality $x + 1 > 8/(x - 1)$?
Let's create a table trying the integers from −5 to 5:

x	***x* + 1**	**8/(*x* – 1)**
−5	−4	−8/6 = −4/3
−4	−3	−8/5
−3	−2	−8/4 = −2
−2	−1	−8/3
−1	0	−8/2 = −4
0	1	−8/1 = −8
1	2	8/0 = undefined
2	3	8/1 = 8
3	4	8/2 = 4
4	5	8/3
5	6	8/4 = 2

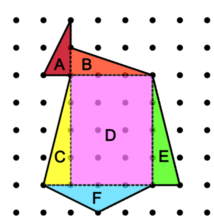
This information is represented on the number line below:



Consider the 10 intervals from −5 to 5. Only 6 of these 10 intervals consist of values that satisfy the given inequality. That's, 6/10 = 60% **Ans.**

Target Round

1. The distance between any two adjacent dots is one unit. Find the area of the shaded polygon.



The shaded polygon has been divided into 6 smaller pieces. The area of each:
A: (1/2) × 2 × 1 = 1
B: (1/2) × 3 × 1 = 1.5
C: (1/2) × 4 × 1 = 2

D: $3 \times 4 = 12$
E: $(1/2) \times 4 \times 1 = 2$
F: $(1/2) \times 4 \times 1 = 2$
Total area: $1 + 1.5 + 2 + 12 + 2 + 2 = 20.5$ **Ans.**

2. A mug is filled with a mixture that is 15 mL of hot chocolate and 35 mL of cream. What percent of the mixture is hot chocolate?
   There are 15 + 35 = 50 mL in the entire mixture. So, hot chocolate accounts for 15/50 = 3/10 = 30% **Ans.**

3. A lottery has 20 million combinations. How many tickets would you need to purchase each second to buy all 20 million combinations in 1 week?
   There are $60 \times 60 \times 24 \times 7 = 604{,}800$ seconds in one week. So, we have $20{,}000{,}000 \div 604{,}800 = 33.06878 \approx 33$ **Ans.**

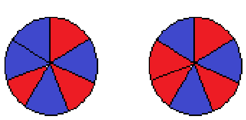
4. Find the mean of all possible positive three-digit integers in which no digit is repeated and all digits are prime.
   Prime digits are 2, 3, 5 and 7.
   There are 3! = 6 combinations of {2,3,5}, {2, 3, 7}, {2, 5, 7} and {3, 5, 7}.
   Starting with 2 we have:
   235, 253, 237, 273, 257 and 273.
   Starting with 3 we have:
   325, 352, 327, 372, 357 and 375.
   Starting with 5 we have:
   523, 532, 527, 572, 537 and 573.
   Starting with 7 we have:
   723, 732, 725, 752, 735 and 753.
   If we were to add up all the numbers, the units column would contain 6 2s, 6 3s, 6 5s and 6 7s.
   That's $(2 \times 6) + (3 \times 6) + (5 \times 6) + (7 \times 6) = 12 + 18 + 30 + 42 = 102$. That gives us a 2 in the units column, and we carry 10 over to the tens column. In the tens column you also have 6 of each number. So that's a sum of 102 plus the 10 we carried over, or 112. That means a 2 in the tens column and we carry over 11. Similarly, there are 6 of each number in the hundreds column. That's a sum of 102 plus the 11 we carried over, or 113.
   The 3 is in the hundreds column so the sum is actually 11,322. The mean is 11,322/24 = 471.75 **Ans.**

5. Ray's age is half his sister's age. Ray's age is also the square root of one-third of his grandfather's age. In 5 years, Ray will be two-thirds as old as his sister will be. We need to find the ratio of Ray's sister's age to his grandfather's age.
   Let r, s and g represent Ray's, Ray's sister's and Ray's grandfather's ages, respectively. Now let's create the equations. We have
   $r = (1/2)s$
   $r = \sqrt{(1/3)g}$
   $r + 5 = (2/3)(s + 5)$
   $(1/2)s + 5 = (2/3)s + 10/3$
   Multiply both sides by 6 yields
   $3s + 30 = 4s + 20$
   $s = 10$
   $r = 5$
   $r^2 = 25 = (1/3)g$
   $g = 25 \times 3 = 75$
   $s/g = 10/75 = 2/15$ **Ans.**

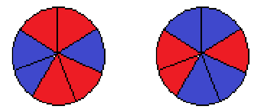
6. Alex added the page numbers of a book and got a total of 888. But there is a page missing. We must find the page number on the final page.
   The sum of the pages numbers is $1 + 2 + \ldots + n - 1 + n = 888$. Since the sum of the consecutive integers from 1 to n is $(n/2) \times (n + 1)$, we can write $(n/2) \times (n + 1) = 888$. This isn't entirely accurate because 888 doesn't include the two

page numbers from the missing sheet. Multiplying by 2 to get rid of the fractions, we get $n(n + 1) = 1776$, and $n^2 + n - 1776 = 0$. We could try and solve this, but remember, the sum isn't accurate.
What this does show us is that the difference between the two roots is 1.
Notice that $\sqrt{1776} \approx 42.1426$. So, let's look at the sum of the first 42 numbers.
We have $(42/2) \times 43 = 21 \times 43 = 903$, and $903 - 888 = 15$. Well, $15 = 8 + 7$ so that must be the page that is missing.
Therefore, the last page must be 42.
42 **Ans.**

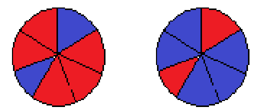
7. A circular spinner has 7 sections of equal size. Each is colored either red or blue. In how many ways can the spinner be colored?
Where you have to be careful is understanding that if you use an odd number of switches (from red to blue or vice versa) the first and last groups of sections are next to each other so the first and last groups are no longer distinct. So we're looking for even numbers of integers that sum to 7 with the exception of all 7 sections being one color.
Let's start with 6 different groups or:
1 + 2 + 1 + 1 + 1 + 1
There are 2 versions (where the red and blue sections are switched).



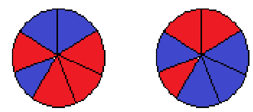
There are no other combinations of 6 groups so let's consider 4 groups..
1 + 2 + 2 + 2



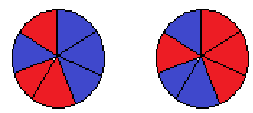
And 1 + 2 + 1 + 3



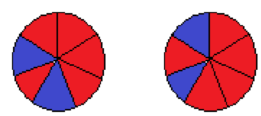
And 1 + 1 + 2 + 3



And 1 + 3 + 2 + 1



And 4 + 1 + 1 + 1



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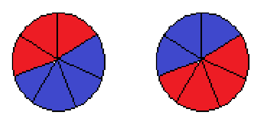
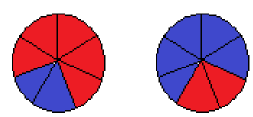
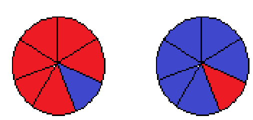
Now two groups: 4 + 3



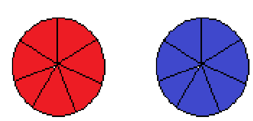
5 + 2.



6 + 1

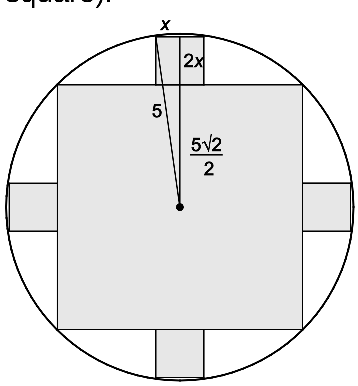


And finally all of the same color:



That's a total of 20. **Ans.**

8. A square is inscribed in a circle of radius 5 units. There are smaller squares, one in each of the regions bounded by a side of the square and the circle as shown. We must find the area of the large square and the four smaller squares.
First, let's determine the side of the large square. If we draw a diagonal in the square, that is the diameter of the circle, or 10. Let s represent the side length of the large square. Based on the properties of 45-45-90 right triangles, we know that $10 = s\sqrt{2}$; thus, $s = 5\sqrt{2}$.
Now, let's draw a right triangle whose long leg and hypotenuse originate at the center of the circle, one extending to the midpoint of the edge of one of the small squares, as shown, and one extending to a point of the same small square that is on the circle. The short leg is a segment connecting the other two (which is half of the side of the small square).



Let $2x$ represent the side length of the small square. The length of the long leg is half the length of the large square plus the length of the small square or $(5\sqrt{2})/2 + 2x$. The length of the hypotenuse is just the radius of the circle, or 5. The length of the short leg, which is half the length of a side of the small square, is x. Using the Pythagorean Theorem, we have

$x^2 + [(5\sqrt{2})/2 + 2x]^2 = 5^2$

$x^2 + 4x^2 + (10\sqrt{2})x + 25/2 = 25$

$5x^2 + (10\sqrt{2})x + 25/2 = 25$

$10x^2 + (20\sqrt{2})x + 25 = 50$

$10x^2 + (20\sqrt{2})x - 25 = 0$

$2x^2 + (4\sqrt{2})x - 5 = 0$

Let's use the quadratic equation to solve this.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 2$, $b = 4\sqrt{2}$ and $c = -5$. We have

$$x = \frac{-4\sqrt{2} \pm \sqrt{(4\sqrt{2})^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-4\sqrt{2} \pm \sqrt{32 + 40}}{4} = \frac{-4\sqrt{2} \pm 6\sqrt{2}}{4}$$

$$x = \frac{3}{2}\sqrt{2} - \sqrt{2} = \frac{\sqrt{2}}{2}$$

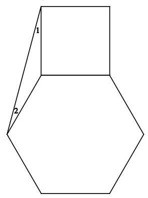
So, $2x = 2 \times \frac{\sqrt{2}}{2} = \sqrt{2}$, and the area of each small square is $(\sqrt{2})^2 = 2$. There are 4 of these squares for a total area of $4 \times 2 = 8$. The area of the large square with side length $5\sqrt{2}$ is $(5\sqrt{2})^2 = 50$. Therefore, the total area of the five squares is $50 + 8 = 58$ **Ans.**

Team Round

1. Greenland is 840,000 square miles. Iceland is only 39,800 square miles. Greenland is 3 times the size of Texas. So what percent of Texas would be covered by Iceland?
$840{,}000 \div 3 = 280{,}000$

39,800 ÷ 280,000 ≈ 14.2% Ans.

2. A square and a regular hexagon share a common side. Find the sum of the degree measures of angles 1 and 2.



Let x and y represent the measures of angles 1 and 2, respectively. Let z represent the measure of the third angle of that triangle, so $x + y = 180 - z$. In a regular hexagon, each interior angle is 120°. Adjacent to the hexagon we have an angle of 90°. That means
$z = 360 - (120 + 90) = 360 - 210 = 150$.
So, $x + y = 180 - 150 = 30$ **Ans.**

3. A trip costs $9000 for the bus plus $125 per student. Each student pays $375. We must find how many students must go on the trip so that the total amount paid is equal to the total cost of the trip. Let x represent the number of students who must go on the trip. Then
$9000 + 125x = 375x$
$250x = 9000$
$x = 36$ **Ans.**

4. Triangle ABC has vertices at A(−3, 4), B(5, 0) and C(1, −4). What is the x-coordinate of the point where the median from C intersects AB?
Since $[5 - (-3)]/2 = 8/2 = 4$, the x-coordinate is $-3 + 4 = 1$ **Ans.**

5. The sum of three primes is 125. The difference between the largest and smallest prime is 50. We must find the largest possible median of these three prime numbers.
OK. Let's list all the primes up to 125.
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
37, 41, 43, 47, 53, 57, 61, 67, 71, 73
79, 83, 89, 97, 101, 103, 107, 109, 113
Let's make a list of pairs of primes that are 50 apart. We have {3, 53}, {11, 61}, {17, 67}, {23, 73}, {29, 79}, {47, 97}, {53, 103}, {57, 107}. Some of these choices have a sum greater than 125. Eliminating those leaves us with {3, 53}, {11, 61}, {17, 67}, {23, 73}, (29, 79}.
Now let's determine which of these two primes will result in a third prime to make them all sum up to 125.
3 + 53 = 56; 125 – 56 = 69; too large and not a prime
11 + 61 = 72; 125 – 72 = 53 which is a prime
17 + 67 = 84; 125 – 84 = 41 which is a prime
23 + 73 = 96; 125 – 96 = 29 which is a prime
29 + 79 = 108; 125 – 108 = 17 which is less than 29
So we have choices of {11, 53, 61}, {17, 41, 67} and {23, 29, 73}
The median of the first set is 53. The median of the second set is 41. The median of the third set is 29. The greatest is 53 **Ans.**

6. What is the probability that a randomly selected integer from 1 to 81, inclusive is equal to the product of two one-digit numbers?
Any number less than or equal to 81 is the product of two one-digit numbers except two-digit primes. Let's see how many primes there are from 10 to 81:
11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79
That's a total of 18 primes.
Add to this any other numbers that are multiples of these primes:
11: 22, 33, 44, 55, 66, 77 or 6 more

13: 26, 39, 52, 65, 78 or 5 more
17: 34, 51, 68 or 3 more
19: 38, 57, 76 or 3 more
23: 46, 69 or 2 more
29: 58 or 1 more
31: 62 or 1 more
37: 74 or 1 more
That's a total of 6 + 5 + 3 + 3 + 2 + 1 + 1 + 1 = 22 more. But we're not done yet: There are also a few integers that are multiples of 3 or more digits:
50 = 2 × 5 × 5
60 = 2 × 2 × 3 × 5
70 = 2 × 5 × 7
75 = 3 × 5 × 5
80 = 2 × 2 × 2 × 2 × 5
That's a total of 5 more. That brings the total number of 18 + 22 + 5= 45 numbers between 1 and 81, inclusive that cannot be written as the product of two one-digit numbers. Therefore, there are 81 – 45 = 36 that are. That's a probability of 36/81 = 4/9 <u>**Ans.**</u>

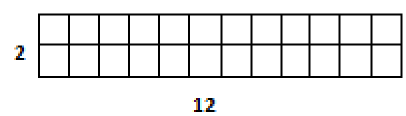
Another way to solve this problem is using the following multiplication table:

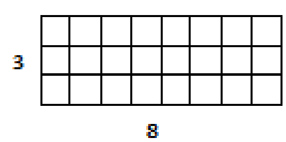
×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

The perfect squares are located on the diagonal. Since any products located below the perfect square in each column will be duplicates, they've been grayed out. Also, 4, 6, 8, 9, 12, 18, 16, 24 and 36 are duplicates, so they've been grayed out as well. What's left are the 36 different numbers from 1 to 81, inclusive, that are the product of two one-digit numbers. Again, that's a probability of 36/81 = 4/9 <u>**Ans.**</u>

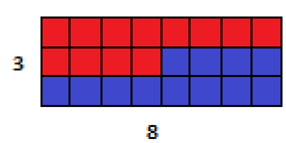
7. We have two sticks, one of length *a* cm and one of length *b* cm. With a stick whose length is strictly between 8 and 26 cm a triangle can be constructed. We must find the product *ab*.
The length of the third side of a triangle is always less than the sum of the lengths of the two other sides. The length of the third side must also be greater than the difference of the lengths of the two sides *a* and *b*. Therefore, $a - b = 8$ and $a + b = 26$. Solving this system by adding the two equations, we get $2a = 24$, and $a = 17$. That means $b = 26 - 17 = 9$. Therefore, $ab = 17 \times 9 = 153$ <u>**Ans.**</u>

8. A 3-inch by 8-inch sheet of paper and a 2-inch by 12-inch sheet of paper have the same area. Using just one cut (not necessarily straight), the 3-inch by 8-inch sheet can be divided into two pieces that can be rearranged to completely cover the 2-inch by 12-inch sheet. Find the length of the cut.



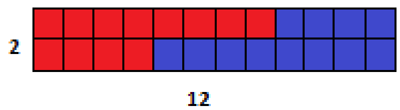


If we cut the 3-by-8 sheet into symmetric "L" shapes, it should work.



Superimposing those cuts onto the

2-by-12 sheet:



The cut is 4 + 1 + 4 = 9 inches. 9 **Ans.**

9. d = 3 + 33 + 303 + 3003 + 30,003 + … Each additional term after the second has one more "interior" 0. The last term has 30 digits. So what is the sum of the digits of d?

$$\begin{array}{r} 3 \\ 33 \\ 303 \\ 3003 \\ 30003 \\ \vdots \\ \underline{+\,300\cdots\cdots\cdots 03} \end{array}$$

Adding the 30 3s in the units column we get 3 × 30 = 90. So, d has a 0 in the units column, and we carry the 9 to the tens column. In the tens column, the sum of the 3 and 28 0s, plus the 9 that was carried is 12. So, d has a 2 in the tens column, and we carry the 1 to the hundreds column. Now in the hundreds column the sum of the 3 and 27 0s, plus the one that was carried is 4. So, d has a 4 in the hundreds column. The sum in each of the remaining 27 columns is 3. Therefore, the sum of the digits of d is (27 × 3) + 4 + 2 + 0 = 87 **Ans.**

10. A bullet train travels 210 km/h. A freight train travels 90 km/h on a parallel track. From the time the bullet train catches up to the back of the freight train to the time the back of the bullet train pulls even with the front of the freight train 24 seconds elapse. Freight trains are three times as long as bullet trains. So how many seconds would it take two bullet trains, each traveling at 210 km/h to pass by each other completely, when moving in opposite directions?
A bullet train that travels 210 km/h travels 210/(60 × 60) = 7/120 km/sec.
A freight train that travels 90 km/h travels 90/(60 × 60) = 3/120 km/sec.
So, when the bullet train starts approaching the freight train to pass, it does so at a rate of (7/120) – (3/120) = 1/30 km/sec. It takes 24 seconds for the back of the bullet train to pull even with the front of the freight train. This means that the front of the bullet train passes the entire freight train and has continued on the full length of the bullet train.
Let f and b represent the lengths of the freight train and bullet train, respectively. We have $f = 3b$. The number of kilometers that the bullet train travels when its rear has pulled even with the front of the freight train is 24 × (1/30) = 4/5 km. So,
$f + b = 4/5$
$3b + b = 4/5$
$4b = 4/5$
$b = 1/5$ km
Given that the two bullet trains are "nose to nose", every second the front of the one bullet train moves (7/120) × 2 = 7/60 km towards the back of the other bullet train. It will take (1/5)/(7/60) = (1/5) × (60/7) = 12/7 seconds for the front of the first bullet train to reach the back of the second bullet train. It will take another 12/7 seconds for the end of the first bullet train to reach the end of the second bullet train. The time for the two bullet trains to completely pass one another is (12/7) + (12/7) = 24/7 = 3.42857 ≈ 3.43 **Ans.**