MATHCOUNTS®

2015
■ Chapter Competition ■
Countdown Round
Problems 1–80

This booklet contains problems to be used in the Countdown Round.

1. _____ When $(x^6y^5z^3)^2$ is simplified, what is the sum of the exponents?

2. _____ (sides) Each exterior angle of a regular polygon measures 24°. How many sides does the polygon have?

3. ($) _____ The total amount Edgar paid for a slice of pizza and a tip of exactly 24% was between $2.50 and $3.00. What was the price of the pizza slice?

4. _____ What is the probability that flipping a fair coin 15 times will yield equal numbers of heads and tails?

5. _____ What prime number is a factor of every four-digit palindrome?

6. _____ (integers) How many positive integers that contain each of the four digits 3, 4, 5 and 7 exactly once are multiples of 4?

7. _____ What is the value of $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\frac{1}{5^2-1}+\frac{1}{6^2-1}$? Express your answer as a common fraction.

8. _____ What is the median of the data in the stem-and-leaf plot, shown here, where 3|9 represents 39?

3	9 9
4	0 3 5 6
5	0 3 7 7 7
6	8 8 9 9
7	4 5

9. _____ (pounds) A wholesaler mixes cashews, almonds and filberts in the ratio 2:3:4, respectively, by weight. How many pounds of almonds will be needed to make 540 pounds of this mixture?

10. _____ What is the sum of the odd numbers between 100 and 200?

11. _____ (codes) How many unique, six-character codes can be made using each of the characters A, B, C, 1, 2 and 3 exactly once?

12. _____ (percent) This weekend, there is a 30% chance it will rain Saturday and a 40% chance it will rain Sunday. If these are independent events, what is the percent chance it will rain at least one day during this weekend?

13. _____ (pounds) Sweet Delights Candy Company has fixed costs of $300. Each pound of candy costs $1 to produce and is sold for $3. How many pounds of candy must be sold so that the company has no profit and no loss?

14. _____ (ft/s) Bob ran the first 1000 feet of a race in 250 seconds and the other 4000 feet in 750 seconds. What was his average speed, in feet per second, for the entire race?

15. _____ If $n\star$ is defined as the product of all even factors of $2n$, for all integers n, where $n > 0$, what is the value of $11\star$?

16. _____ (points) During the first third of the basketball season, Katrina scored an average of 12 points per game. What is the average number of points she must score per game for the remaining two thirds of the season so that her average points scored per game for the entire season is 14 points?

17. _____ If the positive integers 15, 9, 5, 7 and x have a mean and median that are identical, what is the value of x?

18. _____ If a and b both are whole numbers greater than 1, and $\frac{1}{a}+\frac{1}{2a}=\frac{1}{b}$, what is the smallest possible value for b?

19. _____ What is the value of $\sqrt{10}\left(\sqrt{3}+\sqrt{10}\right)$? Express your answer to the nearest whole number.

20. _____ What is the units digit of the product $1 \times 3 \times 5 \times \cdots \times 2015$?

21. _____ What is the greatest possible product of three distinct positive integers that have a sum of 15?

22. _____ (people) On average, Diane makes 20 deliveries for a restaurant in a six-hour shift. What is the minimum number of people, each making deliveries at the same rate as Diane, needed to make 20 deliveries for the restaurant per hour?

23. _____ If $x + y = 5$ and $x^2 + y^2 = 20$, what is the value of xy? Express your answer as a common fraction.

24. _____ (values) How many rational values of x are not integers and satisfy the following equation: $x^7 - 6x^6 + 5x^5 - 4x^4 + 3x^3 - 2x^2 + 1 = 0$?

25. _____ If $4n$ is subtracted from 48 and the difference then is divided by $2n$, the result is 10. What is the value of n?

26. _____ If the mean of the six integers 6, 2, 10, 5, 12 and y is 7, what is the value of y?

27. _____ (marks) The marks on a certain ruler are evenly spaced $\frac{1}{16}$ inch apart. The numbers on this ruler are evenly spaced 1 inch apart. How many marks on this ruler are strictly between the 2-inch mark and the 5-inch mark?

28. _____ If k represents the result when the sum of the first 30 positive odd integers is reduced by 1, what is the sum of the prime factors of k?

29. _____ Edward is one of six people who each are writing 180 math problems. When he solves every problem, he gets an incorrect answer for 10% of the problems that he wrote and for 5% of the problems written by the others. For what fraction of the problems does Edward get the wrong answer? Express your answer as a common fraction.

30. _____ What is the value of $\left(\sqrt{8} + \sqrt{18}\right)^2$?

31. _____ What is the sum of all the integer values of x that satisfy the following two conditions: $|x| < 4$ and $-x < 2$?

32. _____ When Colby rolls two fair standard dice, what is the probability that he rolls the same number on both dice or that the sum of the two numbers rolled is divisible by 3? Express your answer as a common fraction.

33. _____ Circle O has its center at $(-4, 1)$ and a radius of 5 units. What is the sum of the y-coordinates of the two points where circle O intersects the y-axis?

34. _____ When building a staircase, a builder uses the equation $57x - 95y = 0$ to represent the relationship between the height of each step, y, and the depth of each step, x. What is the slope of the staircase? Express your answer as a common fraction.

35. _____ The sum of one-fourth and five-eighths is equivalent to what common fraction?

36. _____ (integers) How many integers that contain each of the four digits 3, 5, 7 and 9 exactly once are prime?

37. _____ What is the remainder when the sum $2015^3 + 2015^2 + 2015^1 + 2015^0$ is divided by 5?

38. _____ What is the greatest common divisor of 4! and 5!?

39. _____ (dollars) A driver switches car insurance companies and saves 15%, which results in a savings of $450. How many dollars does the new insurance policy cost?

40. _____ (integers) In one board game, each player has a unique 4×4 grid with squares randomly labeled with each integer from 1 to 16. As the integers 1 to 16 are randomly called, each player puts an "X" in the square containing that integer. The first player with an "X" in all four squares in any row, column or diagonal wins. At most, how many integers must be called to get a winner?

41. _____ (dollars) If $1000 is to be divided among the first-, second- and third-place prizes in the ratio 9:7:4, how many dollars is the second-place prize?

42. _____ If $3^{2017} \times 9^{2014} = n^{2015}$, what is the value of n?

43. _____ Jon found $\frac{2}{3}$ of an apple pie in the refrigerator. If Jon splits the pie equally between himself and two friends, what fraction of the pie will Jon get? Express your answer as a common fraction.

44. _____ In Antonio's office building, there are nine floors, and the number of steps between consecutive floors is constant. Beginning on the first floor, Antonio walks up the stairs to the ninth floor. When Antonio reaches the third floor, what fraction of his walk from the first to the ninth floor will he have completed? Express your answer as a common fraction.

45. _____ What is the value of $243^{\frac{3}{5}}$?

46. _____ What is the value of $\sqrt{676}$?

47. _____ What is the sum of the distinct positive divisors of 1024?

48. _____ What common fraction is equivalent to $\frac{5! + 4!}{5! - 4!}$?

49. _____ What is the absolute difference between the greatest and least integers that are solutions to $|3x - 7| \leq 8$?

50. _____ What is the value of the quotient $\left(\frac{1/2}{3/4}\right) \div \left(\frac{5/6}{7/8}\right)$ expressed as a common fraction?

51. _____ What is the value of $60^2 - 50^2$?

52. _____ If $27^{x-2} = 729$, what is the value of x?

53. _____ (cm^2) In square centimeters, what is the area of a right triangle with a leg and a hypotenuse of lengths 14 cm and 50 cm, respectively?

54. _____ (students) With one student per seat and no seats left empty, all of the 8th-grade students at Marshall Middle School can fill all the seats on 4 buses and 5 vans. The same students also can fill all the seats on 3 buses and 8 vans. If each van holds 16 students, how many students are in the 8th grade at Marshall Middle School?

55. _____ What is the smallest integer that can be written as a sum of two distinct primes in two distinct ways? Note that 2 + 3 and 3 + 2 are not considered distinct sums.

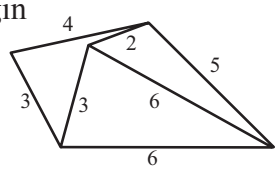
56. _____ On a coordinate plane, D is the image of C reflected about the y-axis. C is the image of B reflected about the x-axis. B is the image of A(3, 2) translated right two units and down five units. What is the sum of the coordinates of D?

57. _____ (minutes) A car is traveling at a uniform rate of 60 mi/h. How many minutes after the car passes highway mile marker 180 will it pass highway mile marker 222?

58. _____ For positive integers a, b and c, with $a > b > c$, the sum $a + b + c$ has the same value as the product $a \times b \times c$. What is the value of $a - b - c$?

59. _____ (degrees) Parallel lines m and n are cut by transversal l. If $\angle$A and $\angle$H are alternate exterior angles and $\angle$A has measure 75°, what is the sum of the degree measures of the complements to $\angle$A and $\angle$H?

60. _____ (miles) The graph shows the distances in miles between the five points on a delivery route. If the delivery person is free to choose where to begin and end the route, how many miles long is the shortest route possible to make all five deliveries?



61. _____ (dollars) A certain website charges each of its advertisers according to the number of monthly visitors to the website. The monthly rate for one of its advertisers is 0.003 cents per visitor. At that rate, if the website received 200,000 visitors during one month, what was that advertiser's monthly charge, in dollars?

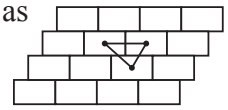
62. _____ (trips) Jennie needs to carry 78 boxes from the cafeteria to the gym. She carries one box on the first trip, two boxes on the second trip, and on each trip after that, Jennie carries one more box than she carried on her previous trip. After how many trips will Jennie first have carried over half of the boxes?

63. _____ One of the 21 dots on a standard die is randomly chosen and colored red. Then the die is rolled. What is the probability that the red dot appears on top? Express your answer as a common fraction.

64. _____ In a regular pentagon, each angle measures $2x$ degrees. What is the value of x?

65. _____ For what value of k will the line $3y + kx = 140$ contain the point $(-5, -5)$?

66. _____ (cm^2) The figure is made from identical rectangles each having area 50 cm^2. The centers of three adjacent rectangles are joined to form a triangle as shown. What is the area of this triangle, in square centimeters?



67. _____ (m^2) In square meters, what is the area of a square with diagonal length $2\sqrt{21}$ meters?

68. _____ What is the smallest positive integer that is divisible by at least four of the numbers in the set {5, 6, 7, 8, 9, 10}?

69. _____ What is the units digit of 2015^{2015}?

70. _____ If positive integers a and b have a greatest common factor of 6 and $25 < a < b < 40$, what is the value of $a + b$?

71. _____ When 48 is added to an integer n, the result is the same as when n is multiplied by 4. What is the value of n?

72. _____ (dollars) If gas is \$3.50 per gallon, the total cost of the gas used to drive 350 miles in a car that can travel 25 miles on a gallon of gas is how many more dollars than the total cost of the gas used to drive the same distance in a car that can travel 35 miles on a gallon of gas?

73. _____ (miles) During a certain week, Joaquin ran 2.5 miles on each of the first 3 days and he ran 3 miles on each of the next 3 days. How many miles must he run on the seventh day to average 3 miles per day for the entire week?

74. _____ (dominoes) In a double-nine domino set, each domino has one number from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} on each half. Every possible combination of numbers appears exactly once, including those that have the same number on each half. How many dominoes are in a double-nine set?

75. _____ What is the least positive integer with exactly 13 positive divisors?

76. _____ (digits) What is the greatest number of digits in the repeating part of any of the decimal representations of $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{10}{11}, \frac{11}{12}, \frac{12}{13}, \frac{13}{14}$ and $\frac{14}{15}$?

77. _____ (cans) A box of cans has 32 rows of 28 cans each. How many cans are in the box?

78. _____ If $x^2 - 14x = -49$, what is the value of $-3x + 10$?

79. _____ (dollars) At Oops Shippers, an envelope is 60¢ and shipping is 40¢ per ounce of contents. What is the cost, in dollars, to ship an envelope with contents weighing a pound?

80. _____ (calories) The number of calories burned while running increases proportionally with the weight of the runner. Two people are running at the same speed. During their half-hour run, the 120-pound person burns 256 calories. How many calories will the 180-pound person burn during this run?