MATHCOUNTS®

2015
■ School Competition ■
Sprint Round
Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

0 1 2 3 4 5 6 7 8 9

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1. _____ If $3x + 2 = 17$, what is the value of x?

2. $ _____ A mountain bike, originally priced at $650, is on sale for 20% off. After the fixed shipping cost of $25 is added to the new price, what is the final cost of the bike?

3. _____ cm Point C divides line segment AB so that $\frac{AC}{AB} = \frac{1}{3}$. If AC = 5 cm, what is AB?

4. _____ games

Cooke County schools have 7 teams playing in a basketball tournament. How many games will be played if each team plays 3 games against every other team?

5. _____ ft^2 What is the area of the largest rectangular field that can be completely enclosed using 120 feet of fencing?

6. _____ The quotient of two positive integers is 21. The difference between the larger integer and this quotient is 21. What is the sum of the two integers?

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7. _____ in^2

A sheet of paper 11 inches tall and $8\frac{1}{2}$ inches wide has 1-inch margins at the top and bottom and $\frac{3}{4}$-inch margins at each side. If nothing is to be printed in the margins, what is the area of the printable region?

8. _____

What is the least positive integer n such that the product $65n$ is a perfect square?

9. _____ years old

The sum of the ages of Kishia and her daughter is 48 years, and the difference of their ages is 20 years. How old is Kishia?

10. _____

A certain cube has volume n cubic inches and surface area n square inches. What is the value of n?

11. _____ seconds

Andrea is pouring water from a full bucket into an empty bucket. The full bucket initially contained 4800 ml of water, and Andrea pours at a steady rate of 100 ml per second. For how many seconds must Andrea pour to get the same amount of water in each bucket?

12. _____ hours

According to a survey, 20 adults in Los Angeles watch an average of 3 hours of television per day, and 40 adults in San Francisco watch an average of 2 hours of television per day. On average, how many hours of television do the 60 adults surveyed watch per day? Express your answer as a mixed number.

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13. _____ unit cubes

Twenty-seven unpainted unit cubes are used to construct a large $3 \times 3 \times 3$ cube. If five of the six faces of the large cube are then painted, how many of the unit cubes are painted on exactly two faces?

14. _____

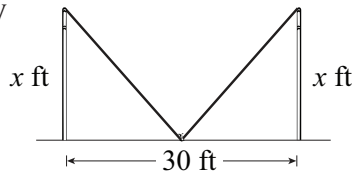
The sequence A_n is such that $a_n = 2a_{n-1} + 1$. If $a_6 = 191$ and $a_5 = 95$, what is the value of a_2?

15. _____

The mean of five whole numbers is 12, and the unique mode is 13. If all five numbers are greater than 9 but less than 14, what is the median of the five numbers?

16. _____

Two vertical poles are placed 30 feet apart, and the top of each pole is x feet above ground. The ends of a wire 34 feet in length are attached at the tops of the poles and its center is anchored to the ground halfway between the poles, as shown. What is the value of x?



17. _____ minutes

A relay team consisting of four runners – Anne, Bryn, Carmen and Dahlia – completed a race in 72 minutes. Anne's leg of the race took $\frac{1}{5}$ of the total time; Bryn's took $\frac{1}{3}$ of the total time; and Carmen's took $\frac{3}{10}$ of the total time. How many minutes did Dahlia take to run her leg of the race?

18. _____

If $ab = 1$, $bc = -9$, $c = -b$ and $c < -a$, what is the value of $a + b$? Express your answer as a mixed number.

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19. _____ coins

Chad has $1.80 in coins consisting of nickels and dimes only. If the total value of his nickels is twice that of his dimes, how many coins does Chad have?

20. _____ ounces

A small bag of 10 identical apples weighs n ounces, and a large bag of n of these apples weighs less than 40 ounces, where n is an integer. In ounces, what is the greatest possible weight of the large bag? Express your answer as a mixed number.

21. _____

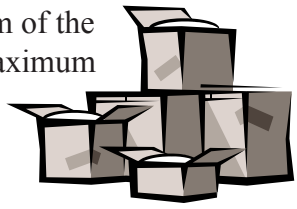
The sum of the first $n + 2$ counting numbers is 43 more than the sum of the first n counting numbers. What is the value of n?

22. _____

What positive integer y satisfies $3x^2 + y^2 - 16 = 3(x^2 + 3)$?

23. _____ in^3

An internet sales company will ship a box for free if the sum of the length, width and height is 48 inches or less. What is the maximum volume of a box 24 inches in length that ships for free ?



24. _____

What is the sum of the first 200 terms of the arithmetic sequence 5, 12, 19, 26, …?

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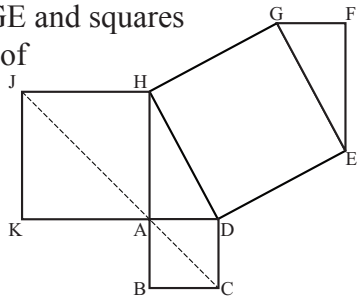
25. _____ mi/h

Maria starts out jogging from Ye Olde Bridge at 8:00 a.m. at a speed of 4 mi/h. Twelve minutes later, Nicole starts out from Ye Olde Bridge and follows the same route. At what constant rate in miles per hour must Nicole run to catch Maria at 8:42 a.m.? Express your answer as a decimal to the nearest tenth.

26. _____

The figure shows congruent right triangles ADH and FGE and squares AHJK, ABCD and DEGH. What is the ratio of the area of quadrilateral CDHJ to the area of hexagon ADEFGH? Express your answer as a common fraction.



27. _____ mg

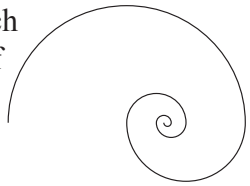
Jose took 500 mg of medication at 2:00 p.m., and then he took another 500 mg at 6:00 p.m. However, 80% of the medication is cleared from his body every 2 hours. How many milligrams of medication remain in his body at 8:00 p.m.?

28. _____

Given the function $f(x) = 6x^2$, what is the value of $\dfrac{f(a+b)-f(a-b)}{ab}$, when a and b are both positive?

29. _____ units

The figure shows the beginning of a spiral created by starting with a semicircle of radius 1 unit and endlessly attaching semicircles that are each half the radius of the previous semicircle. What is the length of the whole spiral? Express your answer in terms of π.



30. _____ cm^2

Right triangle ABC has legs AB = 6 cm and AC = 8 cm. Square BCDE is drawn outside of the triangle along the hypotenuse. What is the area of triangle ADE?

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