
MATHCOUNTS®

2015
■ **State Competition** ■
Countdown Round
Problems 1–80

**This booklet contains problems to be used
in the Countdown Round.**

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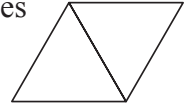
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1. _____ (percent) The number $\frac{4}{5}$ is what percent of $\frac{5}{6}$?

2. _____ If $a > b > 1$ and $a^m b^n = (a^{-2} b^5)(a^6 b^{-2})$, what is the value of the product mn ?

3. _____ The average of a , b and c is 60, and the average of a and b is 90. What is the value of c ?

4. _____ (cm) Two equilateral triangles of side length 6 cm share a side as shown. In centimeters, what is the radius of the smallest circle that encloses both triangles? Express your answer in simplest radical form.



5. _____ (degrees) In the seven-pointed star shown here, what is the mean degree measure of the seven vertex angles? Express your answer as a common fraction.



6. _____ (\$) Mary and Fred purchased sandwiches for a luncheon. Mary spent \$29.95 on 3 medium sandwiches and 2 large sandwiches. Fred spent \$28.45 on 4 medium sandwiches and 1 large sandwich. What would it cost to buy 1 medium sandwich and 1 large sandwich?

7. _____ (ft/s) A ball thrown upward from a height of 4 feet with an initial velocity of v feet per second will be $vt - 16t^2 + 4$ feet above the ground after t seconds. What is the minimum value of v for which the ball reaches a height of at least 20 feet?

8. _____ If the sum of the first 42 positive odd numbers is reduced by 1, what is the sum of the prime factors of the result?

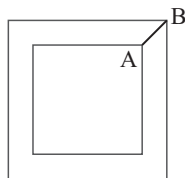
9. _____ (days) A 24-hour clock that runs fast gains 90 seconds every hour. If it is set correctly at 9:00 a.m., in how many days will it next show the correct time?

10. _____ What is the greatest whole number that is a factor of both 2012 and 2013?

11. _____ (inches) The area of a rectangular picture is 96 in^2 , and its width is two-thirds of its height. What is the sum of its height and width, in inches?

12. _____ (units²) What is the smallest possible area, in square units, of a right triangle with two sides of lengths 21 units and 29 units?

13. _____ If $3^x - 1 = 5$, what is the value of $9^x - 1$?
14. _____ (integers) How many positive four-digit integers consist of four digits that are consecutive integers in increasing or decreasing order?
15. _____ What is the sum of the distinct prime factors of 180?
16. _____ (percent) A price increase of 5% followed by a price increase of 6% is equivalent to a single price increase of what percent? Express your answer as a decimal to the nearest tenth.
17. _____ The mean of a set of n numbers is 27, and the mean of another set of $2n$ numbers is 30. What is the mean of a third set of $3n$ numbers if the mean of all $6n$ numbers is 32?
18. _____ What is the sum of the two real numbers x such that $3x^2 - 4x + 5 = \pi^2$? Express your answer as a common fraction.
19. _____ (degrees) If an acute triangle has angles with distinct integer degree measures, what is the least possible degree measure of the smallest angle?
20. _____ What is the product of all numbers x such that $x^3 + 2x^2 - 3x - 6 = 0$ or $6x^3 - 3x^2 - 2x - 1 = 0$?
21. _____ What is the value of $x + x^2 + x^4 + x^8$ when $x = 2$?
22. _____ (inches)



A 6" by 6" photo is surrounded by a 1" wide frame. In inches, what is the length of segment AB, which joins a corner of the photo to a corner of the frame? Express your answer in simplest radical form.

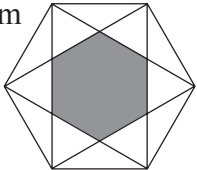
23. _____ What is the mean of the 17 numbers represented in this stem-and-leaf plot, where $3|9 = 39$?

3	9 9
4	1 3 5 6
5	2 3 5 7 8
6	4 5 7 9
7	1 1

24. _____ (participants) A summer camp had 78 participants who played at least one of soccer, basketball and volleyball. Of these participants, 35 played soccer, 30 played basketball, 27 played volleyball, 7 played both basketball and volleyball, 5 played both volleyball and soccer, 6 played both soccer and basketball. How many participants played all three sports?
25. _____ The product of two nonnegative integers is 10,000. If neither number is a multiple of ten, what is the sum of those two integers?
26. _____ (laps) When Maria has run one-fourth of her third lap in a race, she will have completed one-fourth of the entire race. How many laps are in the race?
27. _____ If $x^2 + y^2 = 12$ and $x^2 - y^2 = 5$, for $x > 0$ and $y > 0$, what is the value of the product xy ? Express your answer as a common fraction in simplest radical form.
28. _____ (units²) In square units, what is the area of the region bounded by the graphs of $y = |x|$ and $y = \sqrt{16 - x^2}$? Express your answer in terms of π .
29. _____ If the integers 1, 2, 3 and 4 are each used exactly once to replace the variables a , b , c and d , what is the greatest possible value of $(a + b)^{c + d}$?
30. _____ (people) How many people must be in a group to guarantee that at least three people in the group were born in the same month?
31. _____ (inches) The diagonals of a rhombus have lengths 15 inches and 36 inches. What is the perimeter of the rhombus, in inches?
32. _____ (cards) In the game of *Swick*, each card contains a letter from A to J, inclusive, followed by an integer from 0 to 9, inclusive. Each possible letter-number combination appears on exactly one card. How many cards are in a *Swick* deck?
33. _____ (units²) A rectangle has an area of 10 units² and a diagonal of length 11 units. What is the sum of the squares of the rectangle's length and width?
34. _____ What is the absolute difference between 3.45×10^6 and 1.35×10^4 ? Express your answer to the nearest whole number.

35. _____ What is the sum of the odd positive divisors of 10,000?
36. _____ (percent) If 20% of x is equal to 30% of y , what percent of y is x ?
37. _____ The terms of an increasing arithmetic sequence have a common difference of 3. If the sum of four consecutive terms is 72, what is the sum of the next four consecutive terms?
38. _____ If $f(x) = x + 2$ and $g(x) = 3x - 2$, what is the value of $g(f(5))$?
39. _____ (seconds) The height y , in feet, of a soccer ball x seconds after it is kicked is given by the equation $y = 40x - 16x^2$. How many seconds after the ball is kicked will it first hit the ground? Express your answer as a decimal to the nearest tenth.
40. _____ (units) What is the shortest distance from the line $y = x + 1$ to the origin? Express your answer as a common fraction in simplest radical form.
41. _____ If $x:y = 2:3$ and $y:z = 4:5$, what is the value of $\frac{x}{z}$? Express your answer as a common fraction.
42. _____ Two real numbers are randomly chosen strictly between 0 and 1. What is the probability that their sum exceeds their product? Express your answer as a decimal to the nearest tenth.
43. _____ What is the value of $(5^{2015} + 5^{-2015})^2 - (5^{2015} - 5^{-2015})^2$?
44. _____ If a number written in base 2 consists entirely of a sequence of 2015 1s, what is the units digit of the number in base 10?
45. _____ If a 45° arc on circle O has the same length as a 120° arc on circle P, what is the ratio of the area of circle O to the area of circle P? Express your answer as a common fraction.
46. _____ What common fraction is equivalent to $\overline{0.45}$?

47. _____ (percent) Sandra bought a bag of jelly beans. She sorted the jelly beans and found $\frac{3}{10}$ were yellow, $\frac{2}{5}$ were green and $\frac{1}{10}$ were red. The remaining jelly beans were purple. What percent of the jelly beans were purple?
48. _____ When rolling five dice, what is the probability that they will all show even numbers or all show odd numbers? Express your answer as a common fraction.
49. _____ If x is an integer such that $(x + 3)(x + 4) = 42$, what is the least possible value of $(x + 4)(x + 5)$?
50. _____ If $\frac{(4!)!}{4!} = a!$, what is the value of a ?
51. _____ The variables a, b, c, d and e represent five different positive integers. The median, mean and mode of $\{a, a, b, c, d, e\}$ (which includes a twice) are the same. What integer is the smallest possible sum of the six numbers?
52. _____ Max rolls a standard die twice. What is the probability that the first number he rolls is no larger than the second number? Express your answer as a common fraction.
53. _____ If n is a positive integer greater than 2 and $\sqrt[n]{n} = \sqrt{2}$, what is the value of n ?
54. _____ If the decimal representation of the common fraction $\frac{a}{b}$ begins with 0.13 followed by at least one more nonzero digit, what is the minimum possible value of b ?
55. _____ (integers) How many positive integers are a multiple of 1000 and a factor of 1,000,000?
56. _____ What is the sum of the positive divisors of 256?
57. _____ If $a > b > 0$, what is the simplest form of the expression $\frac{a - \frac{b^2}{a}}{b - \frac{a^2}{b}}$? Express your answer as a common fraction in terms of a and b .

58. _____ What is the only integer between -100 and 100 that has more than 12 positive divisors?
59. _____ What is the smallest prime number that is not a divisor of $14!$?
60. _____ Each term of the sequence $3, 10, 17, 24, \dots$, is seven more than the term that precedes it. What is the value of the term of the sequence that is closest to 2015?
61. _____ What is the value of x if $\frac{2}{x-14} = \frac{3}{4x}$? Express your answer as a decimal to the nearest tenth.
62. _____ For what value of x does $8^{(9^{10})} = 2^{(3^x)}$?
63. _____ For positive integers a and b , $a + b = 20$. What is the greatest possible value of $a^2 + b^2$?
64. _____ (cm²) In a regular hexagon of area 120 cm^2 , six diagonals are drawn to form a smaller, shaded hexagon as shown. What is the area of the smaller hexagon, in square centimeters?
- 
65. _____ (cm) The area of a sector of a circle is $12\pi \text{ cm}^2$. The measure of the central angle of the sector is 120° . In centimeters, what is the radius of the circle?
66. _____ (percent) Park Middle School has 80 girls and 74 boys in the 7th grade. It has 96 girls and 70 boys in the 8th grade. What percent of the 7th- and 8th-grade students at Park Middle School are girls?
67. _____ (positive integers) How many positive integers are a factor of $2^8 - 1$?
68. _____ What is the least positive integer with exactly 16 positive factors?
69. _____ (amounts) How many amounts from 1¢ to 99¢ , inclusive, can be created using only dimes and quarters?

70. _____ Ryan squared a positive number, then added $\frac{1}{9}$ of the result, then increased its value by 10, and finally decreased the result by 20%. All of these steps gave a result of 208. What was Ryan's original number?

71. _____ (percent) If the length of each edge of a cube is increased by 20%, by what percent does the volume increase? Express your answer to the nearest whole number.

72. _____ (percent) A band sells the rights to one of its songs for a certain amount of money. After the band receives 60% of the money, the manager gets 25% of what remains. The record company then receives 20% of what remains after the band and its manager have been paid. The rest goes to the shareholders. What percent of the original amount will the shareholders receive?

73. _____ What is the value of $(4 + 6 + 8 + 10 + 12)^3$?

74. _____ For what integer x does $2^x = 4^x$?

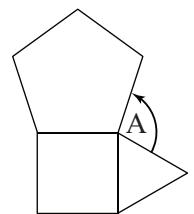
75. _____ (students) The ratio of the number of 6th, 7th and 8th grade students at King Middle School is 9:5:7, respectively. If there are a total of 630 students at King, how many of the students are in the 8th grade?

76. _____ What is the value of $155^2 - 145^2 + 135^2 - 125^2$?

77. _____ If $f(x) = 2x^2 + 8$, what is the positive value of x for which $f(x) = 136$?

78. _____ For what integer n is the value of 2^n closest to 1,000,000?

79. _____ (degrees) Three regular polygons have common sides and vertices, as shown. What is the degree measure of angle A?



80. _____ (IDs) At Matrix Middle School student IDs start with a 7 or 8 to indicate the grade level, followed by an F or M for gender, followed by a 1 or 0 to indicate whether the student is a bus rider or not, followed by a three-digit number that does not contain the digits 0, 1, 7 or 8. How many distinct student IDs are possible?