
MATHCOUNTS®

2016
■ State Competition ■
Countdown Round
Problems 1–80

This booklet contains problems to be used
in the Countdown Round.

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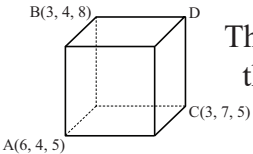
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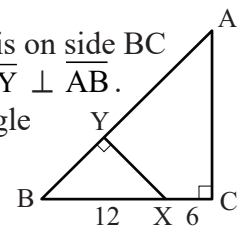
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1. _____ (factors) How many positive integer factors does 2016 have?
2. _____ If x is a positive number such that $x\%$ of x is 289, what is the value of x ?
3. _____ (meters) Quadrilaterals ABCD and WXYZ are similar. The area of quadrilateral ABCD is five times that of quadrilateral WXYZ. If $BD = 10$ meters, what is XZ , in meters? Express your answer in simplest radical form.
4. _____ The six-digit number $31A,B2B$, where A and B represent distinct digits, is divisible by 225. What is the value of A ?
5. _____ What is the value of $\frac{37,037}{7007}$? Express your answer as a mixed number.
6. _____ If a number is increased by $p\%$, and then the new number is decreased by $p\%$, the result is 1% less than the original number. What is the value of p ?
7. _____ Given that $w - 3 = x + 5 = y - 7 = z + 9$, what is the absolute difference between the least and the greatest of the numbers w , x , y and z ?
8. _____ What is the units digit of the sum $4^1 + 4^2 + 4^3 + \dots + 4^{2016}$?
9. _____ (factors) If $c = 2! \cdot 3! \cdot 4!$, what is the total number of positive integer factors of c ?
10. _____ If $x > 1, y > 1, \frac{(xy^2)^{15}}{(x^2y)^9} = 1$ and $x = y^k$, where $k > 0$, what is the value of k ?
11. _____ Two standard, fair six-sided dice are rolled. If the sum of the numbers rolled is greater than or equal to 8, what is the probability that the sum is odd? Express your answer as a common fraction.
12. _____ (dollars) Tracee purchased two items, each originally priced at \$22.50. However, Tracee got a 12% discount on one item and a 28% discount on the other. What is the combined amount Tracee paid for these two items?
13. _____ (units)



The coordinates of three vertices of a cube are shown. What is the length of the space diagonal with endpoints A and D , in units? Express your answer in simplest radical form.
14. _____ What is the greatest 4-digit integer whose digits all are prime and for which the sum of the digits is prime?
15. _____ If $d = 123456^3 - 123^5$, what is the least prime factor of d ?

16. _____ (palindromes) Excluding the numbers in which all three digits are the same, how many 3-digit positive integer palindromes are there?
17. _____ (mm) Triangle ABC has integer side lengths and its perimeter is 18 mm. If $AB = AC$, what length of side BC, in millimeters, minimizes the area of triangle ABC?
18. _____ (values) How many integer values of x satisfy the inequality $|2x + 1| \leq |3 - x|$?
19. _____ (minutes) Each day, Todd rows 12 miles upriver and 12 miles back. His speed in still water is 9 mi/h. On Monday, the current was 1 mi/h, and on Tuesday the current was 3 mi/h. What is the absolute difference between the number of minutes Todd spent rowing on the two days?
20. _____ What is the value of $\left(x + \frac{1}{x}\right)^2$, if $x = \sqrt{\frac{5}{8}}$? Express your answer as a common fraction.
21. _____ If $6^4 \cdot 8^2 = 2^a \cdot 9^b$ for whole numbers a and b , what is the value of $a + b$?
22. _____ (inches) Two rectangles are similar. The smaller rectangle has length 9 inches and width 5 inches. The larger rectangle has length $4x - 3$ inches and width $2x + 2$ inches. What is the perimeter of the larger rectangle, in inches?
23. _____ (days) On the first day of a two-week reading project, Dirk spent 45 minutes reading. On each subsequent day, he read for 15 more minutes than he did the day before. On how many days did Dirk spend less than 10% of a 24-hour day reading?
24. _____ (miles) Susan's odometer displays 15,951 miles, a palindrome. What is the minimum number of miles she must drive before it displays another palindrome?
25. _____ The sum of three integers is 80. The ratio of the first integer to the second is 3:4, and the ratio of the second integer to the third is 4:9. What is the absolute difference between the greatest and least of these integers?
26. _____ In isosceles right triangle ABC, shown here, $AC = BC$. Point X is on side BC such that $CX = 6$ and $XB = 12$, and Y is on side AB such that $\overline{XY} \perp \overline{AB}$. What is the ratio of the area of triangle BXY to the area of triangle ABC? Express your answer as a common fraction.
27. _____ For any 3-digit number ABC, the 6-digit number ABC,ABC is always divisible by a 1-digit number greater than 1. What is that 1-digit number?

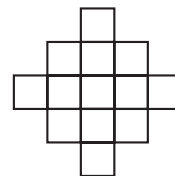


28. _____ (dollars) A half-dollar coin has the same weight as five dimes. Three pounds of half-dollar coins are worth \$60. How many dollars are three pounds of dimes worth?

29. _____ What is the sum of all positive integers k such that the equation $x^2 + 11x + k = 0$ has two integer solutions?

30. _____ What is the least integer greater than 1,000,000 that is divisible by 18?

31. _____ Denis chooses two different unit squares at random from the array of 13 unit squares shown. What is the probability that the two squares share an edge? Express your answer as a common fraction.



32. _____ A circle of diameter 3 inches is inside a circle of diameter 5 inches. What is the probability that a dart that lands inside the larger circle does not land inside the smaller circle? Express your answer as a common fraction.

33. _____ (minutes) A subway train arrives at a station at midnight on Sunday. An additional 240 trains arrive at the same station by midnight the following Sunday, with the last one arriving exactly one week after the original train. What is the average number of minutes between train arrivals at this station?

34. _____ What is the sum of the three missing digits in the subtraction problem $5\square,661 - \square2,83\square = 17,825$?

35. _____ (digits) How many digits are in the product $8^9 \cdot 5^{24}$?

36. _____ What is the value of the expression $\frac{20.16^2 - 1}{19.16}$? Express your answer as a decimal to the nearest hundredth.

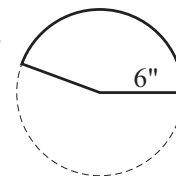
37. _____ (units²) The diameter and height of a closed cylinder are equal. If the volume of the cylinder is 128π units³, what is its surface area, in square units? Express your answer in terms of π .

38. _____ A 3-digit number, for which each digit is 1, 4 or 7, with repetition allowed, is chosen at random. What is the probability that the sum of the three digits is 12? Express your answer as a common fraction.

39. _____ If $x > 0$ and $\sqrt{3} \cdot \sqrt{7} \cdot \sqrt{10} \cdot \sqrt{x} = x$, what is the value of x ?

40. _____ What is the 7th term of the geometric sequence $9, 6, 4, \frac{8}{3}, \dots$? Express your answer as a common fraction.

41. _____ (inches) The lateral surface of a cone is constructed by rolling up a 160° sector cut from a circle of radius 6 inches. What is the radius of the base of this cone, in inches? Express your answer as a common fraction.



42. _____ (units²) On the coordinate plane, the five vertices of a pentagon are (0, 0), (4, 0), (6, 3), (4, 5) and (0, 3). What is the area of the pentagon, in square units?

43. _____ Given that $(2^{n+1})^2 = \frac{4^{2n}}{8^n}$, what is the value of n ?

44. _____ The integer 273 can be written as a sum of seven consecutive positive integers. What is the sum of the two greatest of these integers?

45. _____ (cm) A ball of copper in the shape of a sphere with radius 3 cm is reshaped into a cylindrical wire with radius 0.06 cm. What is the length, in centimeters, of the wire?

46. _____ If $\frac{3}{8}$ of $\frac{2}{7}$ of a number is 1 more than $\frac{3}{28}$ of $\frac{1}{8}$ of that same number, what is the number? Express your answer as a common fraction.

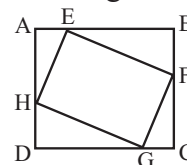
47. _____ (marbles) Ten green marbles are removed from a bag, leaving twice as many green marbles as red marbles. Fifteen red marbles are then removed from the bag, leaving five times as many green marbles as red marbles. How many green marbles were originally in the bag?

48. _____ What is the sum of all integer values of n between 75 and 100 such that n divided by 6 has remainder 5 and n divided by 4 has remainder 3?

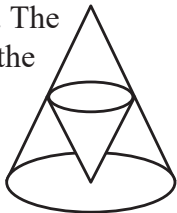
49. _____ (seconds) Mike and Mark start at opposite ends of a straight 100-yard track. They repeatedly run to the opposite end, turn around and run back. They run at the same constant speed, start at the same time, and pass each other for the first time after 7 seconds. How many seconds have elapsed when they pass each other for the fifth time?

50. _____ (ft²) What is the area, in square feet, of a square with diagonal length 6 feet?

51. _____ (in²) Rectangle EFGH is inscribed in rectangle ABCD so that the four similar right triangles are formed, as shown. If $m\angle BEF = 30^\circ$, $EF = 12$ inches and $FG = 2\sqrt{3}$ inches, what is the area of rectangle ABCD, in square inches? Express your answer in simplest radical form.



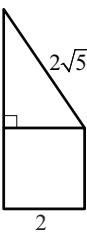
52. _____ What is the sum of the composite positive integers less than 100 that are not divisible by 2, 3 or 5?

53. _____ The equation $x^3 - 3x^2 + ax + b = 0$ has 1 and -2 as two of its three solutions. What is the value of $a + b$?
54. _____ If a positive 2-digit number is chosen at random, what is the probability that its digits differ by 1? Express your answer as a common fraction.
55. _____ What is the fifth digit to the right of the decimal point in the value of the difference $0.213 - 0.123$?
56. _____ In a particular sequence, the first term is 20, the second term is 16, and each subsequent term is the mean of all preceding terms. What is the 2016th term in the sequence?
57. _____ The set of all the positive 4-digit integers that can be formed using each of the digits 1, 2, 3 and 7 exactly once are arranged in increasing order. What is the 13th number?
58. _____ When Yin attempts to add all the positive integers from 1 through 99, he accidentally skips a number and gets the sum 4904. Which number did Yin skip?
59. _____ A small circular cone is inscribed in a large circular cone, as shown. The apex of the small cone is the center of the base of the large cone. If the base radius of the small cone measures half that of the large cone, what is the ratio of the volume of the small cone to the volume of the large cone? Express your answer as a common fraction.
- 
60. _____ What is the greatest integer less than 100 that cannot be written as a sum of two primes?
61. _____ What is the greatest proper divisor of $391^2 - 389^2$?
62. _____ If x and y are perfect squares such that $x + y = 65$ and $x > y$, what is the least possible value of $x - y$?
63. _____ (students) If the students at Conformity Middle School all weigh exactly 112 pounds each, how many of these students can safely stand on a bridge with a 10,000-pound weight limit?
64. _____ (units²) What is the area, in square units, of a rectangle with diagonal length 15 units and width 5 units? Express your answer in simplest radical form.

65. _____ (sculptures) Five ounces of clay are required for Theresa to complete each sculpture. What is the maximum number of sculptures she can complete using 15 pounds of clay?

66. _____ A bag contains 7 red tiles and 3 blue tiles. If two tiles are drawn from the bag at random, without replacement, what is the probability that the tiles will be different colors? Express your answer as a common fraction.

67. _____ (games) The Mudcats baseball team currently has 12 wins and 12 losses. What is the fewest number of consecutive games they must win to have won at least 70% of their games?

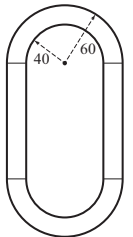
68. _____ (units²)  A square with sides of length 2 units is constructed on a right triangle with a hypotenuse of length $2\sqrt{5}$ units to form the figure shown. What is the total area of the figure, in square units?

69. _____ (times) On a 12-hour digital clock, sometimes all of the digits are the same (for example, 3:33). How many times during a 24-hour day are all of the digits the same?

70. _____ The reflection of $P(4, -5)$ across the line $x = 1$ is P' . If the reflection of P' across the line $y = 2$ is $P''(a, b)$, what is the value of $a + b$?

71. _____ The dimensions of a 9-inch by 12-inch rectangular picture are reduced by $x\%$. Then the dimensions of the smaller picture are reduced by $x\%$, resulting in a picture that measures 2.25 inches by 3 inches. What is the value of x ?

72. _____ (feet) A running track consists of two straight regions and two semicircular regions as shown. The inner and outer radii of the semicircles at each end are 40 feet and 60 feet, respectively. The straight parts are each 120 feet long. What is the absolute difference, in feet, between the inner and outer perimeters of the track? Express your answer in terms of π .



73. _____ In the sequence 37, 45, 53, 61, ..., each number after the first term is 8 greater than the previous term. What is the least integer greater than 500 in the sequence?

74. _____ If the value of an expression written in the form $a + \sqrt{b}$ is six less than half its square, what is the greatest value of $a + b$?

75. _____ When the decimal number $0.\overline{1234}$ is written as a common fraction, what is the sum of the numerator and denominator?

76. _____ (passwords) Samantha needs to choose a password with 6 distinct characters. The first two characters must be digits from 0 to 4, inclusive. The second two are lowercase letters of the alphabet, and the last two are digits from 5 to 9, inclusive. How many possible passwords can Samantha make?
77. _____ What is the sum of the solutions to $x^2 + 3x = 10$?
78. _____ (hours) The time in Sydney, Australia is 17 hours ahead of the time in Portland, Oregon. If it is 5:00 p.m. in Portland, how many hours after midnight is it in Sydney?
79. _____ Mr. Dexter has 30 desks in his classroom numbered 1 through 30. If Ken randomly sits at a desk, what is the probability that he will sit at a desk with a number that is not prime? Express your answer as a common fraction.
80. _____ If $\frac{3}{5 + \frac{1}{x}} = \frac{1}{4}$, what is the value of x ? Express your answer as a common fraction.