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# MATHCOUNTS®

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2016  
■ State Competition ■  
Sprint Round  
Problems 1–30

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1
2
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5
6
7
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## HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature \_\_\_\_\_ Date \_\_\_\_\_

Printed Name \_\_\_\_\_

School \_\_\_\_\_

Chapter \_\_\_\_\_

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## DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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Total Correct	Scorer's Initials

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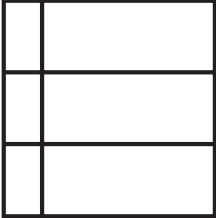
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1. \_\_\_\_\_ Let  $a @ b = \frac{a}{2a+b}$ . What is the value of  $5 @ 3$ ? Express your answer as a common fraction.

2. \_\_\_\_\_ rectangles
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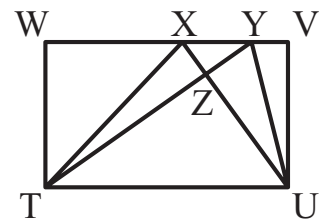
How many rectangles of any size are in the grid shown here?

3. \_\_\_\_\_ Given  $7x + 13 = 328$ , what is the value of  $14x + 13$ ?

4. \_\_\_\_\_ What is the median of the positive perfect squares less than 250?

5. \_\_\_\_\_ If  $\frac{x+5}{x-2} = \frac{2}{3}$ , what is the value of  $x$ ?

6. \_\_\_\_\_ units<sup>2</sup> In rectangle TUVW, shown here,  $WX = 4$  units,  $XY = 2$  units,  $YV = 1$  unit and  $UV = 6$  units. What is the absolute difference between the areas of triangles TXZ and UYZ?



7.            green  
           marbles A bag contains 4 blue, 5 green and 3 red marbles. How many green marbles must be added to the bag so that 75 percent of the marbles are green?

8.            miles MD rides a three wheeled motorcycle called a trike. MD has a spare tire for his trike and wants to occasionally swap out his tires so that all four will have been used for the same distance as he drives 25,000 miles. How many miles will each tire drive?



9.            years Lucy and her father share the same birthday. When Lucy turned 15 her father turned 3 times her age. On their birthday this year, Lucy's father turned exactly twice as old as she turned. How old did Lucy turn this year?

10.            The sum of three distinct 2-digit primes is 53. Two of the primes have a units digit of 3, and the other prime has a units digit of 7. What is the greatest of the three primes?

11.            pounds Ross and Max have a combined weight of 184 pounds. Ross and Seth have a combined weight of 197 pounds. Max and Seth have a combined weight of 189 pounds. How many pounds does Ross weigh?

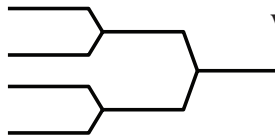
12.            What is the least possible denominator of a positive rational number whose repeating decimal representation is  $0.\overline{AB}$ , where A and B are distinct digits?

13. \_\_\_\_\_ miles A taxi charges \$3.25 for the first mile and \$0.45 for each additional  $\frac{1}{4}$  mile thereafter. At most, how many miles can a passenger travel using \$13.60? Express your answer as a mixed number.

14. \_\_\_\_\_ percent Kali is mixing soil for a container garden. If she mixes  $2 \text{ m}^3$  of soil containing 35% sand with  $6 \text{ m}^3$  of soil containing 15% sand, what percent of the new mixture is sand?

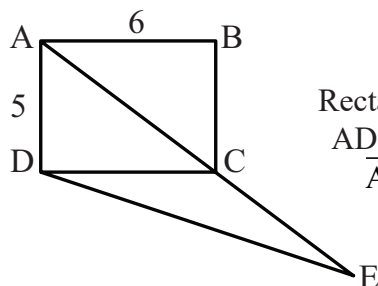
15. \_\_\_\_\_ laps Alex can run a complete lap around the school track in 1 minute, 28 seconds, and Becky can run a complete lap in 1 minute, 16 seconds. If they begin running at the same time and location, how many complete laps will Alex have run when Becky passes him for the first time?

16. \_\_\_\_\_ The Beavers, Ducks, Platypuses and Narwhals are the only four basketball teams remaining in a single-elimination tournament. Each round consists of the teams playing in pairs with the winner of each game continuing to the next round. If the teams are randomly paired and each has an equal probability of winning any game, what is the probability that the Ducks and the Beavers will play each other in one of the two rounds? Express your answer as a common fraction.



17. \_\_\_\_\_ A function  $f(x)$  is defined for all positive integers. If  $f(a) + f(b) = f(ab)$  for any two positive integers  $a$  and  $b$  and  $f(3) = 5$ , what is  $f(27)$ ?

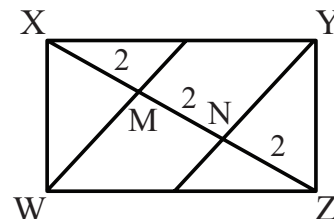
18. \_\_\_\_\_ units



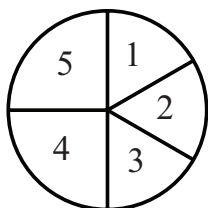
Rectangle ABCD is shown with  $AB = 6$  units and  $AD = 5$  units. If  $\overline{AC}$  is extended to point E such that  $\overline{AC}$  is congruent to  $\overline{CE}$ , what is the length of  $\overline{DE}$ ?

19. \_\_\_\_\_ The digits of a 3-digit integer are reversed to form a new integer of greater value. The product of this new integer and the original integer is 91,567. What is the new integer?

20. \_\_\_\_\_ units<sup>2</sup> Diagonal XZ of rectangle WXYZ is divided into three segments each of length 2 units by points M and N as shown. Segments MW and NY are parallel and are both perpendicular to XZ. What is the area of WXYZ? Express your answer in simplest radical form.

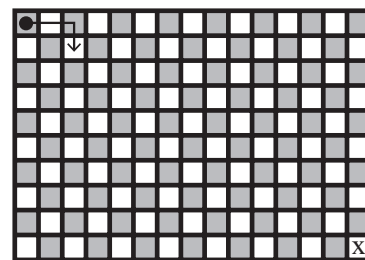


21. \_\_\_\_\_ A spinner is divided into 5 sectors as shown. Each of the central angles of sectors 1 through 3 measures  $60^\circ$  while each of the central angles of sectors 4 and 5 measures  $90^\circ$ . If the spinner is spun twice, what is the probability that at least one spin lands on an even number? Express your answer as a common fraction.



22. \_\_\_\_\_ orders The student council at Round Junior High School has eight members who meet at a circular table. If the four officers must sit together in any order, how many distinguishable circular seating orders are possible? Two seating orders are distinguishable if one is not a rotation of the other.

23. \_\_\_\_\_ moves Initially, a chip is placed in the upper-left corner square of a  $15 \times 10$  grid of squares as shown. The chip can move in an L-shaped pattern, moving two squares in one direction (up, right, down or left) and then moving one square in a corresponding perpendicular direction. What is the minimum number of L-shaped moves needed to move the chip from its initial location to the square marked "X"?



24. \_\_\_\_\_ units On line segment AE, shown here, B is the midpoint of segment AC and D is the midpoint of segment CE. If  $AD = 17$  units and  $BE = 21$  units, what is the length of segment AE? Express your answer as a common fraction.



25. \_\_\_\_\_ There are twelve different mixed numbers that can be created by substituting three of the numbers 1, 2, 3 and 5 for  $a$ ,  $b$  and  $c$  in the expression  $a\frac{b}{c}$ , where  $b < c$ . What is the mean of these twelve mixed numbers? Express your answer as a mixed number.
26. \_\_\_\_\_ If 738 consecutive integers are added together, where the 178th number in the sequence is 4,256,815, what is the remainder when this sum is divided by 6?
27. \_\_\_\_\_ points Consider a coordinate plane with the points  $A(-5, 0)$  and  $B(5, 0)$ . For how many points  $X$  in the plane is it true that  $XA$  and  $XB$  are both positive integer distances, each less than or equal to 10?
28. \_\_\_\_\_ The function  $f(n) = a \cdot n! + b$ , where  $a$  and  $b$  are positive integers, is defined for all positive integers. If the range of  $f$  contains two numbers that differ by 20, what is the least possible value of  $f(1)$ ?
29. \_\_\_\_\_ strings In the list of numbers 1, 2, ..., 9999, the digits 0 through 9 are replaced with the letters A through J, respectively. For example, the number 501 is replaced by the string "FAB" and 8243 is replaced by the string "ICED". The resulting list of 9999 strings is sorted *alphabetically*. How many strings appear before "CHAI" in this list?
30. \_\_\_\_\_ cm A 12-sided game die has the shape of a hexagonal bipyramid, which consists of two pyramids, each with a regular hexagonal base of side length 1 cm and with height 1 cm, glued together along their hexagons. When this game die is rolled and lands on one of its triangular faces, how high off the ground is the opposite face? Express your answer as a common fraction in simplest radical form.

## Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

**Units of measurement are not required in answers, but they must be correct if given.** When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

**All answers must be expressed in simplest form.** A “common fraction” is to be considered a fraction in the form  $\pm \frac{a}{b}$ , where  $a$  and  $b$  are natural numbers and  $\text{GCF}(a, b) = 1$ . In some cases the term “common fraction” is to be considered a fraction in the form  $\frac{A}{B}$ , where  $A$  and  $B$  are algebraic expressions and  $A$  and  $B$  do not share a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form  $\pm N\frac{a}{b}$ , where  $N$ ,  $a$  and  $b$  are natural numbers,  $a < b$  and  $\text{GCF}(a, b) = 1$ . Examples:

*Problem:* What is  $8 \div 12$  expressed as a common fraction?      *Answer:*  $\frac{2}{3}$       *Unacceptable:*  $\frac{4}{6}$

*Problem:* What is  $12 \div 8$  expressed as a common fraction?      *Answer:*  $\frac{3}{2}$       *Unacceptable:*  $\frac{12}{8}$ ,  $1\frac{1}{2}$

*Problem:* What is the sum of the lengths of the radius and the circumference of a circle with diameter  $\frac{1}{4}$  unit expressed as a common fraction in terms of  $\pi$ ?      *Answer:*  $\frac{1+2\pi}{8}$

*Problem:* What is  $20 \div 12$  expressed as a mixed number?      *Answer:*  $1\frac{2}{3}$       *Unacceptable:*  $1\frac{8}{12}$ ,  $\frac{5}{3}$

**Ratios should be expressed as simplified common fractions** unless otherwise specified. Examples:

*Simplified, Acceptable Forms:*  $\frac{7}{2}$ ,  $\frac{3}{\pi}$ ,  $\frac{4-\pi}{6}$       *Unacceptable:*  $3\frac{1}{2}$ ,  $\frac{1}{3}$ , 3.5, 2:1

**Radicals must be simplified.** A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

*Problem:* What is the value of  $\sqrt{15} \times \sqrt{5}$ ?      *Answer:*  $5\sqrt{3}$       *Unacceptable:*  $\sqrt{75}$

**Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$)  $a.bc$ , where  $a$  is an integer and  $b$  and  $c$  are digits.** The *only* exceptions to this rule are when  $a$  is zero, in which case it may be omitted, or when  $b$  and  $c$  both are zero, in which case they both may be omitted. Answers in the form (\$)  $a.bc$  should be rounded to the nearest cent unless otherwise specified. Examples:

*Acceptable:* 2.35, 0.38, .38, 5.00, 5      *Unacceptable:* 4.9, 8.0

**Do not make approximations for numbers** (e.g.,  $\pi$ ,  $\frac{2}{3}$ ,  $5\sqrt{3}$ ) in the data given or in solutions unless the problem says to do so.

**Do not perform any intermediate rounding** (other than the “rounding” a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

**Scientific notation** should be expressed in the form  $a \times 10^n$  where  $a$  is a decimal,  $1 \leq |a| < 10$ , and  $n$  is an integer. Examples:

*Problem:* What is 6895 expressed in scientific notation?      *Answer:*  $6.895 \times 10^3$

*Problem:* What is 40,000 expressed in scientific notation?      *Answer:*  $4 \times 10^4$  or  $4.0 \times 10^4$

**An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.**

Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

**The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.**