
MATHCOUNTS®

2015
■ State Competition ■
Team Round
Problems 1–10

School _____
Chapter _____
Team _____
Members _____, Captain

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

Raytheon

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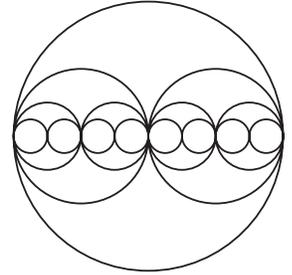
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1. _____ When the integers 1 through 7 are written in base two, what fraction of the digits are 1s? Express your answer as a common fraction.

2. _____ sets Using the figure of 15 circles shown, how many sets of three distinct circles A, B and C are there such that circle A encloses circle B, and circle B encloses circle C?



3. _____ cm^2 Equilateral triangle ABC with side-length 12 cm is inscribed in a circle. What is the area of the largest equilateral triangle that can be drawn with two vertices on segment AB and the third vertex on minor arc AB of the circle? Express your answer in simplest radical form.

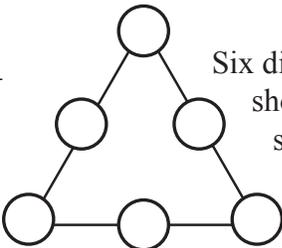
4. _____ minutes Sue and Kara run along the perimeter of a 120-yard by 40-yard rectangular field. In the time it takes Sue to run 120 yards along one side at a rate of 10 minutes per mile, Kara runs the lengths of the other three sides. At Kara's current rate, how many minutes would it take her to run a mile?

5. _____ An arithmetic sequence has first term a and common difference d . If the sum of the first ten terms is half the sum of the next ten terms, what is the ratio $\frac{a}{d}$? Express your answer as a common fraction.

6. _____ students In one math class, all of the students wrote the amount of change in their pockets on the board. Then they computed some properties of these numbers: the median was 50¢, the mean was 40¢ and there was a unique mode of 40¢. What is the fewest number of students that can be in this class?

7. _____ units Four consecutive sides of an equiangular hexagon have lengths of 1, 9, 16 and 4 units, in that order. What is the absolute difference in the lengths of the two remaining sides?

8. _____ convex sequences A *convex sequence* is a sequence of integers where each term (other than the first and last) is no greater than the arithmetic mean of the terms immediately before and after it. For example, the sequence 4, 1, 2, 3 is convex because $1 \leq \frac{4+2}{2}$ and $2 \leq \frac{1+3}{2}$. How many convex sequences use each number in the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ exactly once?

9. _____  Six different prime numbers are placed in the six different circles shown. The three circles on each side of the triangle have the same sum. What is the least possible value of the side sum?

10. _____ Two integers have the property that when their sum is added to the arithmetic mean of their squares, the result is 64. What is the greatest possible value of the product of these two integers?

