
MATHCOUNTS®

2018
■ Chapter Competition ■
Countdown Round
Problems 1–80

**This booklet contains problems to be used
in the Countdown Round.**

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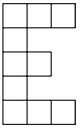
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1. _____ What is the sum of the integers from 1 to 100, inclusive?
2. _____ If $f(x) = 3x + 4$ and $g(x) = 2x - 4$, what is $f(g(-8))$?
3. _____ (points) If two distinct circles and an ellipse are drawn, what is the maximum possible number of points at which at least two of the three curves intersect?
4. _____ (minutes) Mikhail can type 80 words in one minute. At this rate, how many minutes will it take for Mikhail to type 320 words?
5. _____ (games) A total of 32 softball teams play in a single-elimination tournament, meaning the losers of each round are eliminated and the winners of each round are matched up against each other in the next round, until one champion remains. How many total games are played in the tournament?
6. _____ Valerie is exactly as much taller than Philip as Philip is taller than Janelle. If Valerie is 5 feet 6 inches tall, Philip is 4 feet 10 inches tall, and Janelle is 4 feet x inches tall, what is the value of x ?
7. _____ What is the greatest five-digit multiple of 5 with five distinct odd digits?
8. _____ What is the mean of the six numbers shown? Express your answer as a decimal to the nearest hundredth.

632.14	632.12	632.13
632.15	632.16	632.14
9. _____ Peter counts backwards by 3s starting with 100. What is the average of the first five numbers that Peter counts?
10. _____ If $(2^x)^{\frac{1}{2}} = 16$, what is the value of x ?
11. _____ (dollars) A prize of \$900 is to be shared in the ratio of 2:1:1 among three awardees. How many dollars are in the smallest share?
12. _____ (tickets) As a fund raiser, a booster club sells \$10 raffle tickets for a chance to win a free vacation. If the cost to print any number of raffle tickets is \$25, what is the minimum number of raffle tickets that must be sold to raise \$2500?

13. _____ What is the product of the number that is 8 more than 16 and the number that is 12 less than 5?

14. _____ The figure shown is made of ten unit squares. If the figure has area m units² and perimeter n units, what is the value of $m + n$?



15. _____ (home runs) In 2001, Barry led the league with 73 home runs. Alex finished fourth with 52. Sammy and Luis finished second and third, respectively. Sammy hit seven more home runs than Luis. If these four players hit a total of 246 home runs in 2001, how many home runs did Sammy hit that year?

16. _____ What is the least integer for which $3x - 7 > 24$?

17. _____ (hours) In a single week, how many hours pass between 10 p.m. on Monday and 7 a.m. on Wednesday?

18. _____ (eggs) A grocer sells her organic eggs in full containers of 6 or 7 eggs. What is the greatest number of eggs you cannot buy from her?

19. _____ Harry tosses 2 fair coins. What is the probability that at least one coin lands heads up? Express your answer as a common fraction.

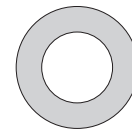
20. _____ (students) A class contains 40 students wearing gloves, scarves, hats or some combination of these. If 30 students are wearing gloves or scarves but not hats and 28 are wearing scarves or hats but not gloves, what is the absolute difference between the number of students wearing gloves and the number of students wearing hats?

21. _____ What is the value of the sum $1^2 + 3 + 4^3 + 6$?

22. _____ What value of n makes the equation $2^5 \times 4^2 = 8^n$ true?

23. _____ A 20-sided die with sides labeled with the letters A through T is rolled. What is the probability the letter facing up is a vowel? Express your answer as a common fraction.

24. _____ (ft²) The two concentric circles shown have radii of 2 feet and 5 feet. In square feet, what is the area of the shaded region between the circles? Express your answer in terms of π .



25. _____ What positive value r satisfies the equation $r = \frac{2}{1+r}$?

26. _____ What is the sum of the coefficients in the polynomial $x^4 + 4x^3 + 6x^2 + 4x + 1$?

27. _____ (dollars) Connie, Nate and Eli each want to buy a sundae. But Connie is 4 dollars short, Nate is 3 dollars short, and Eli is 1 dollar short. Combined, they have exactly enough money to buy one sundae. How many dollars does one sundae cost?

28. _____ What is the positive difference between the sum and product of 17 and 29?

29. _____ (seconds) Water comes out of a hose at a rate of 3 gallons per minute. Using this hose, how many seconds will it take to fill a bucket with 4 gallons of water?

30. _____ (units²) What is the area of a triangle with vertices at (1, 8), (9, 2) and (1, 2)?

31. _____ (dollars) Samantha bought a blouse and a skirt for \$70. The blouse cost \$20 more than the skirt. What was the cost of the blouse, in dollars?

32. _____ (units) The area of a rectangle is 40 units². If the length of the rectangle is 12 units, what is the width, in units? Express your answer as a common fraction.

33. _____ (degrees) The degree measure of the vertex angle of an isosceles triangle is twice the measure of each base angle. What is the degree measure of a base angle?

34. _____ Two standard six-sided dice are tossed. What is the probability that both the numbers shown are prime? Express your answer as a common fraction.

35. _____ What is the greatest integer solution of the inequality $|3x - 5| \leq 12$?

36. _____ What number lies at the midpoint of the segment on the number line given by $|2x + 3| \leq 5$? Express your answer as a common fraction.

37. _____ (unit cubes) A $3\text{-cm} \times 3\text{-cm} \times 3\text{-cm}$ cube is painted blue on all sides. The cube is then cut into unit cubes of edge length 1 cm. How many of the unit cubes have no paint on any side?
38. _____ (dollars) Harry made a one-time deposit of \$2000 for the use of a couch. When he returned the couch five years later, he was refunded \$1000. How many dollars per year did he pay for the use of the couch?
39. _____ If $3x - 7 = 3 + 2y$, what is the value of $9x - 6y$?
40. _____ What is the value of $(4.2 \times 10^{15}) \div (8.4 \times 10^{11})$? Express your answer as a whole number.
41. _____ (miles) Brice rides his bike at a constant speed of 8 mi/h for 15 minutes, then speeds up and rides at a constant speed of 10 mi/h for 30 minutes. During these 45 minutes, how many miles did he travel?
42. _____ (chickens) John has a barnyard containing only goats and chickens. The chickens each have 1 head and 2 legs. The goats each have 1 head and 4 legs. If there are 11 heads and 30 legs in the barnyard, how many chickens are in the barnyard?
43. _____ (buses) A school bus can transport as many as 50 students. What is the minimum number of school buses needed to transport 425 students on a field trip?
44. _____ A math class has 13 girls and 7 boys. If one girl is added to the class, what fraction of the class is now girls? Express your answer as a common fraction.
45. _____ What is the sum of the greatest common factor and least common multiple of the set $\{1, 2, 3, 4, 5\}$?
46. _____ The product of three consecutive odd integers is 315. What is the least of the integers?
47. _____ If $x + 3y = 1$, $2x + 5z = 11$ and $x + y - z = -4$, what is the value of $x + y + z$?

48. _____ (blue marbles) In a bag with 40 marbles, one-fourth are blue. How many blue marbles must be added to this bag so that half the marbles in the bag are blue?
49. _____ (degrees) The degree measures of a pair of complementary angles are in the ratio 1:5. What is the degree measure of the larger angle?
50. _____ (dollars) Joni's stock increased by 20% in the last year. It is now worth \$5 more than a year ago. What was the stock's value a year ago, in dollars?
51. _____ (words) Alex and Thomas take part in a rap battle. Alex raps for 90 seconds at an average word rate of 140 words per minute. Thomas has 75 seconds to respond. How many words does Thomas need to average per minute in his response to match Alex's word count?
52. _____ If $a \& b = a^2 + b^2$, what is the units digit of $((((1 \& 2) \& 3) \& 4) \& 5)$?
53. _____ (in³) A cylinder is filled with water to a depth of 6 inches. More water is added, increasing the depth of the water to 7.5 inches. If the original volume of water was 36 in³, what is the volume, in cubic inches, after more water is added?
54. _____ (degrees) What is the degree measure of an exterior angle of a regular nine-sided polygon?
55. _____ (servings) One serving of hot chocolate requires 2 tablespoons of cocoa, 1 tablespoon of sugar and 1 cup of milk. Milton has 2 cups of cocoa, $\frac{1}{2}$ cup of sugar and 1 gallon of milk. What is the greatest number of servings of hot chocolate Milton can make, given that 16 tablespoons = 1 cup and 16 cups = 1 gallon?
56. _____ A rectangle with non-congruent adjacent sides and a square share a side of length 6. The perimeter of the rectangle is 40. What is the ratio of the area of the square to the area of the rectangle. Express your answer as a common fraction.
57. _____ Eva's average score for seven algebra quizzes is 90, with the highest possible score on any quiz being 100. Eva scored 92 on each of five quizzes. Based on this, what is the lowest score she could have earned on any one of her quizzes?

58. _____ The product of 3 consecutive positive integers is divisible by $6!$. What is the least possible average of the 3 numbers?
59. _____ If $\{a, b, c, d, e\}$ is an increasing arithmetic sequence of five prime numbers, what is the least possible value of $|a - b|$?
60. _____ (percent) Of the 80% of the students in a geography class who earned a passing grade, 20% earned an A. What percent of the students in the entire class earned an A?
61. _____ The circumference of a circle with area π^2 can be expressed as $2\pi^b$. What is the value of b ? Express your answer as a common fraction.
62. _____ Dan rolls four standard six-sided dice. What is the probability that the product of the four numbers rolled is prime? Express your answer as a common fraction.
63. _____ (percent) Clara is in a rush, so she only gets sunscreen on 70% of her body and bug spray on 80%. What is the least possible percent of Clara's body that is protected by both? Express your answer as a whole number.
64. _____ (hours) The number of bacteria A present after t hours is given by $A(t) = 45.5 \times 8^{\frac{t}{12}}$. How many hours does it take for the number of bacteria present to double?
65. _____ (prime numbers) How many two-digit prime numbers are also prime numbers when the digits of the number are reversed?
66. _____ (cm) An equilateral triangle and a regular octagon share a side. If the triangle has a perimeter of 24 cm, what is the perimeter of the octagon, in centimeters?
67. _____ What number is $\frac{5}{6}$ of the way from -57 to 27 ?
68. _____ A standard 52-card deck contains four aces. If two cards are drawn randomly without replacement, what is the probability that they are both aces? Express your answer as a common fraction.
69. _____ (m/s) Jenny takes 90 steps per minute. If each step is 80 cm, what is Jenny's speed in meters per second? Express your answer as a decimal to the nearest tenth.

70. _____ What is the sum of the 11th and the 44th digits to the right of the decimal point in the decimal representation of $\frac{1}{7}$?
71. _____ (shots) In a basketball game, Jeff scored a total of 36 points. Of the shots he made, 20% were worth three points each, 40% were worth two points each and 40% were worth one point each. How many shots worth one point did Jeff make?
72. _____ (sides) The sum of the interior angle measures of a convex polygon is 900 degrees. How many sides does the polygon have?
73. _____ (zeros) How many trailing zeros are there in the decimal representation of $25!$?
74. _____ When Shirley goes bowling, the probability that she gets at least one strike during any given game is $\frac{1}{5}$, and distinct games are independent from one another. If Shirley plays three games, what is the probability that she gets at least one strike? Express your answer as a common fraction.
75. _____ What is the value of ${}_4C_0 + {}_4C_1 - {}_4C_2 + {}_4C_3 + {}_4C_4$?
76. _____ Marcus randomly selects a positive composite number less than 100 that has an odd number of positive factors. What is the probability that his number is also less than 50? Express your answer as a common fraction.
77. _____ (sides) A convex polygon has 35 diagonals. How many sides does it have?
78. _____ If two distinct letters are randomly chosen from the English alphabet of 26 letters, what is the probability that both are in the set {A, E, I, O, U}? Express your answer as a common fraction.
79. _____ (dollars) A \$270,000 prize is to be divided in the ratio of 5:3:1. What is the largest portion in dollars?
80. _____ What is the positive square root of 4×3^6 ?